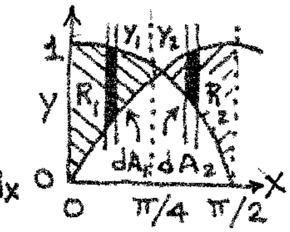
QSci 292 • answers • Hmwk #8

Neu. 3rd ed., sec. 6.3, p 321
Neu. 2nd ed., sec. 6.3.6, p 392



#8) $R_1 = \{(x,y): 0 \le x \le \frac{\pi}{4},\ y_1 \le y \le y_2\}$

$dA_1 = (y_1 - y_2)\,dx = (\cos(x) - \sin(x))\,dx$

$$A_1 = \int_0^{\pi/4} (c - s)\,dx = \left[(s + c\Big|_0^{\pi/4})\right] = \left[\left(\tfrac{1}{\sqrt{2}} + \tfrac{1}{\sqrt{2}}\right) - (0 + 1)\right]$$

$\boxed{A_1 = \sqrt{2} - 1}$ or $A_1 = 0.414$ sq. units

#8) Neu. 2nd ed. $A = A_1 + A_2$ $R_2 = \{(x,y): \frac{\pi}{4} \le x \le \frac{\pi}{2},\ y_2 \le y \le y_1\}$

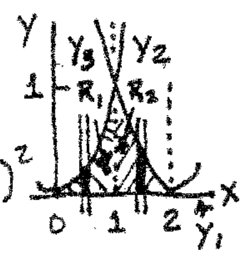
$dA_2 = (y_2 - y_1)\,dx = (\sin(x) - \cos(x))\,dx$ $A_2 = \int_{\pi/4}^{\pi/2} (s - c)\,dx$

$$A_2 = -\left[(c + s\Big|_{\pi/4}^{\pi/2}\right] = -\left[(0 + 1) - \left(\tfrac{1}{\sqrt{2}} + \tfrac{1}{\sqrt{2}}\right)\right]$$

$\boxed{A = 2(\sqrt{2} - 1)}$ $A = 0.828$ sq. units

#10) $R_1 = \{(x,y): 0 \le x \le 1,\ y_1 \le y \le y_2\}$

$R_2 = \{(x,y): 1 \le x \le 2,\ y_1 \le y \le y_3\}$



where : $y_1 = 0$, $y_2 = x^2$, $y_3 = (2 - x)^2$

$dA_1 = x^2\,dx$; $dA_2 = (2 - x)^2\,dx$

$$A_1 = \int_0^1 x^2\,dx = \frac{1}{3}\ ;\ A_2 = \int_1^2 (2 - x)^2 = \frac{1}{3}$$

$\boxed{A = \frac{2}{3}}$ or $A = 0.667$ sq units

note:
at $x = 1$,
$y_2 = y_3$

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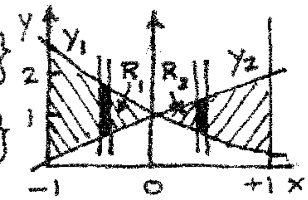
#12) $R_1 = \{(x,y): -1 \le x \le 0,\ y_1 \le y \le y_2\}$

$R_2 = \{(x,y):\ 0 \le x \le 1,\ y_2 \le y \le y_1\}$

with $y_1 = e^{-x}$; $y_2 = 1+x$



$dA_1 = [e^{-x} - (1+x)]\,dx$, $dA_2 = [1+x-e^{-x}]\,dx$

$$A_1 = \int_{-1}^{0} (e^{-x} - 1 - x)\,dx = -\left[\left(e^{-x} + x + \tfrac{1}{2}x^2 \Big|_{-1}^{0}\right)\right]$$

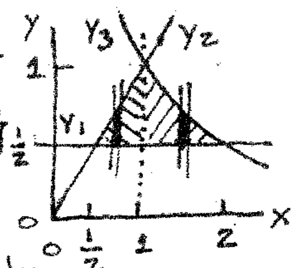
$$A_2 = \int_{0}^{+1} (1 + x - e^{-x})\,dx = \left[x + \tfrac{1}{2}x^2 + e^{-x}\Big|_{0}^{1}\right]$$

$\boxed{A = e + \frac{1}{e} - 1}$ or $A = 2.086$

#14) $R_1 = \{(x,y): \frac{1}{2} \le x \le 1,\ y_1 \le y \le y_2\}$

$R_2 = \{(x,y):\ 1 \le x \le 2,\ y_1 \le y \le y_3\}$

with $y_1 = \frac{1}{2}$, $y_2 = x$, $y_3 = \frac{1}{x}$



$dA_1 = (x - \frac{1}{2})\,dx$, $dA_2 = \left(\frac{1}{x} - \frac{1}{2}\right)dx$

$$A_1 = \int_{1/2}^{1} (x - \tfrac{1}{2})\,dx = \left[\tfrac{x}{2}(x-1)\Big|_{1/2}^{0}\right] = \tfrac{1}{8}$$

$$A_2 = \int_{1}^{2} \left(\tfrac{1}{x} - \tfrac{1}{2}\right)dx = \left[\left(\ln x - \tfrac{1}{2}x\right)\Big|_{1}^{2}\right] = \ln 2 - \tfrac{1}{2}$$

$\boxed{A = \ln 2 - \frac{3}{8}}$ or $A = 0.318$

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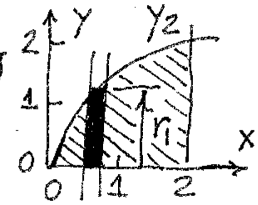
#36) $R = \{(x,y): 0 \le x \le 2,\ y_1 \le y \le y_2\}$

with $y_1 = 0$ and $y_2 = \sqrt{2x}$



$dV = \pi r_1^2\,dx = \pi(\sqrt{2x})^2\,dx$

$V = \pi \int_0^2 2x\,dx$, $\boxed{V = 4\pi}$ or $V = 12.56$ cu. units

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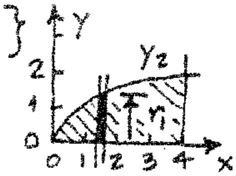
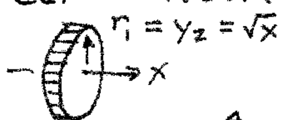
#36) $R = \{(x,y): 0 \le x \le 4,\ y_1 \le y \le y_2\}$

New. 2nd ed.

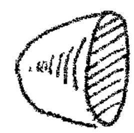
with $y_1 = 0$ and $y_2 = \sqrt{x}$





$dV = \pi r_1^2\,dx = \pi(\sqrt{x})^2\,dx$

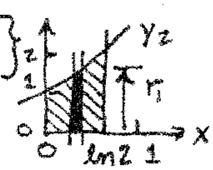
$V = \pi \int_0^4 x\,dx$, $\boxed{V = 2\pi}$ or $V = 6.28$ cu. units



3-D object

#38) $R = \{(x,y): 0 \le x \le \ln 2,\ y_1 \le y \le y_2\}$

with $y_1 = 0$ and $y_2 = e^x$



$dV = \pi r_1^2\,dx = \pi(e^x)^2\,dx$

$V = \pi \int_0^{\ln 2} e^{2x}\,dx = \left[\frac{\pi}{2}\left(e^{2x}\right)\Big|_0^{\ln 2}\right] = \frac{\pi}{2}\left(e^{2\ln 2} - 1\right)$

$\boxed{V = \frac{3}{2}\pi}$ or $V = 4.712$ cubic units

3-D object

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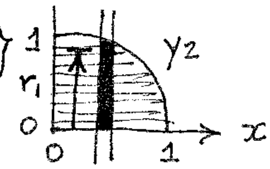
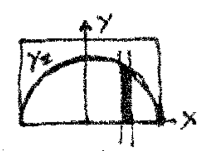
#40) $R = \{(x,y): 0 \le x \le 1,\ y_1 \le y \le y_2\}$
with $y_1 = 0$ & $y_2 = \sqrt{1-x^2}$



$dV = \pi r_1^2\,dx$; $V = \pi \int_0^1 \left(\sqrt{1-x^2}\right)^2 dx$

$V = \pi\left(x - \frac{1}{3}x^3 \Big|_0^1\right)$ $\boxed{V = \frac{2}{3}\pi}$ or $V = 2.09$ cu. units

#40) $R = \{(x,y): -2 \le x \le 2,\ y_1 \le y \le y_2\}$
New. 2nd ed. $y_1 = 0$ and $y_2 = \sqrt{4-x^2}$



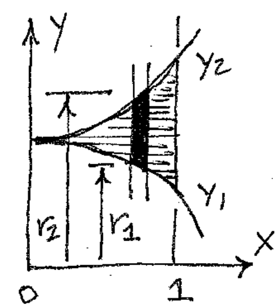
$dV = \pi\,(4-x^2)\,dx$; $V = \pi \int_{-2}^{+2} (4-x^2)\,dx$

semi-circle

$\boxed{V = \frac{32}{3}\pi}$ or $V = 33.5$ cubic units

sphere

#42) $R = \{(x,y): 0 \le x \le 1,\ y_1 \le y \le y_2\}$
with $y_1 = 2 - x^3$ & $y_2 = 2 + x^3$



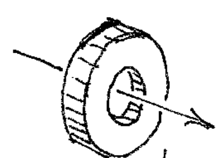
$dV = \pi r_2^2\,dx - \pi r_1^2\,dx$

$dV = \pi\left[(2+x^3)^2 - (2-x^3)^2\right]dx$

$dV = 8\pi x^3\,dx$

$V = 8\pi \int_0^1 x^3\,dx$

$\boxed{V = 2\pi}$ or $V = 6.28$ cu. units



dV
hollow disk

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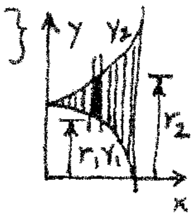
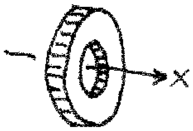
#42) $R = \{(x, y) : 0 \le x \le 1, y_1 \le y \le y_2\}$

Neu. 2nd ed. with $y_1 = 1 - x^2$ and $y_2 = 1 + x^2$





$$dV = \pi r_2^2 \, dx - \pi r_1^2 \, dx$$

$$dV = \pi\left[(1+x^2)^2 - (1-x^2)^2\right] dx = 4\pi x^2 dx$$

$$V = 4\pi \int_0^1 x^2 \, dx, \quad \boxed{V = \frac{4}{3}\pi} \text{ or } V = 4.19 \text{ cu units}$$

3-D cross section

Neu. 3rd ed., ch. 7, rev. prob., p. 387
Neu. 2nd ed., sec. 7.9, rev. prob., p 470

#32) $I = \int_0^{\pi/2} x \sin(x)\, dx$; I.P.

$$F(x) = \sin(x) - x \cdot \cos(x) + C$$

$$I = \left[\sin\left(\tfrac{\pi}{2}\right) - \tfrac{\pi}{2}\cdot\cos\left(\tfrac{\pi}{2}\right)\right] - \left[0 - 0(1)\right]$$

$$\boxed{I = 1}$$

#36) $I = \int_0^{1/2} \frac{2}{\sqrt{1-x^2}}\, dx = 2\left(\arcsin(x)\Big|_0^{1/2}\right)$

$$I = 2\left[\frac{\pi}{6} - 0\right] \text{ or } \boxed{I = \frac{\pi}{3}}$$