QSci 292 • answers • Hmwk #9

Neu. 3rd ed, sec. 7.4, p. 362
Neu. 2nd ed, sec. 7.4, p. 442

#2) $I = \int_0^{\infty} x e^{-x}\, dx$; replace "∞" with M
(M = large number)

$$I_M = \int_0^M x e^{-x}\, dx \; ; \quad F = (x-1)e^{-x} \quad \underline{\underline{\text{I.P.}}}$$

$$I_M = \left[(x-1)e^{-x} \Big|_0^M \right] = (M-1)\frac{1}{e^M} - (0-1)1$$

$$I = \lim_{M\to\infty} \left\{ 1 + (M-1)\frac{1}{e^M} \right\} = 1$$

$\therefore$ I exists and $\boxed{I = 1}$

#9) $I = \int_{-\infty}^{+\infty} \frac{x}{(1+x^2)^2}\, dx$; replace "$\pm\infty$"

$$I_M = \int_{-M}^{+M} \frac{x}{(1+x^2)^2}\, dx \; ; \quad \underline{\text{S.M.}} \quad u = 1+x^2, \quad du = 2x\, dx$$

$$F = \frac{1}{2}\int \frac{1}{u^2}\, du = -\frac{1}{2}\frac{1}{u} = -\frac{1}{2}\frac{1}{1+x^2}$$

$$I = \lim_{M\to\infty} \left\{ -\frac{1}{2}\left[\frac{1}{1+M^2} - \frac{1}{1+(-M)^2} \right] \right\}$$

$\therefore$ I exists and $\boxed{I = 0}$

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#10) $I = \int_{-\infty}^{+\infty} x^3 e^{-x^4}\,dx$; $I_M = \int_{-M}^{+M} x^3 e^{-x^4}\,dx$

$F = -\frac{1}{4} e^{-z} = -\frac{1}{4} e^{-x^4}$; $\underline{\text{S.M.}}$

$z = x^4$

$I_M = -\frac{1}{4}\left[e^{-M^4} - e^{-(-M)^4}\right]$ $dz = 4x^3\,dx$

$I = \lim\limits_{M \to \infty} \{I_M\} = 0$; $\therefore$ I exists; $\boxed{I = 0}$

#18) $I = \int_{1}^{\infty} \frac{1}{x^{1/3}}\,dx$; $I_M = \int_{1}^{M} \frac{1}{x^{1/3}}\,dx$

$F = \frac{3}{2} x^{2/3}$; $I_M = \frac{3}{2}\left(M^{2/3} - 1\right)$

$I = \lim\limits_{M \to \infty}(I_M) \longrightarrow \infty$ $\boxed{\text{I does NOT exist}}$

#28) $I = \int_{-\infty}^{1} \frac{3}{1+x^2}\,dx$; $I_M = 3\int_{-M}^{1} \frac{1}{1+x^2}\,dx$

$I_M = 3\left[\arctan(1) - \arctan(-M)\right]$

$I = \lim\limits_{M \to \infty} \{I_M\} = 3\left[\frac{\pi}{4} - \left(-\frac{\pi}{2}\right)\right]$

$\therefore$ I exists and $\boxed{I = \frac{9\pi}{4} = 7.07}$

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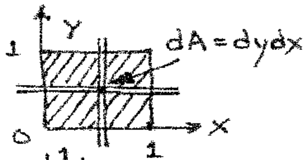
Double Integral Problems: Rectangular Regions (R)

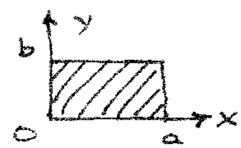
#15) $I = \int_0^1 \int_0^1 (x + y^2)\,dy\,dx$



$$I = \int_0^1 \left(x + \tfrac{1}{3}\right)dx = \left(\tfrac{1}{2}x^2 + \tfrac{1}{3}x \Big|_0^1\right) = \tfrac{1}{2} + \tfrac{1}{3}$$

$\therefore \boxed{I = 5/6}$

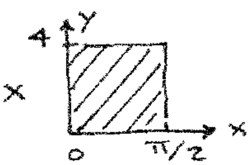
#16) $I = \int_0^a \int_0^b (x^2 + y^2)\,dy\,dx$



$$I = \int_0^a \left(x^2 \cdot b + \tfrac{1}{3}b^3\right)dx = \left(\tfrac{1}{3}x^3 b + \tfrac{1}{3}b^3 x \Big|_0^a\right)$$

$\boxed{I = \tfrac{1}{3}\,ab\,(a^2 + b^2)}$

#17) $I = \int_0^4 \int_0^{\pi/2} x \cdot \cos(y)\,dy\,dx$



$$I = \int_0^4 x\,(1)\,dx = \tfrac{1}{2}x^2 \Big|_0^4$$

$\boxed{I = 8}$

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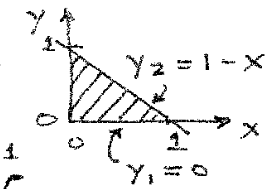
Double Integral Problems: Non-Rectangular Regions (Q)

#2) $I = \int_0^1 \int_{y_1}^{y_2} 2xy \, dy \, dx$

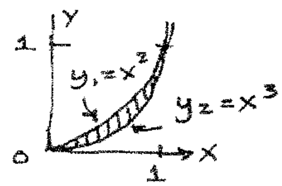


$$I = \int_0^1 2x\left(\tfrac{1}{2}y_2^2 - \tfrac{1}{2}y_1^2\right)dx = \int_0^1 x(1-x)^2 dx$$

$$I = \left(\tfrac{1}{2}x^2 - \tfrac{2}{3}x^3 + \tfrac{1}{4}x^4 \Big|_0^1\right) = \tfrac{1}{2} - \tfrac{2}{3} + \tfrac{1}{4}$$

$$\boxed{I = \frac{1}{12}}$$

#7) $I = \int_0^1 \int_{y_1}^{y_2} x \, dy \, dx$



$$I = \int_0^1 x(y_2 - y_1)dx = \int_0^1 x(x^2 - x^3)dx$$

$$I = \left(\tfrac{1}{4}x^4 - \tfrac{1}{5}x^5 \Big|_0^1\right) = \tfrac{1}{4} - \tfrac{1}{5}$$

$$\boxed{I = \frac{1}{20}}$$