ANSWERS to SAMPLE QUIZ

P1: (1) $F = \int \frac{9x+2}{9x^2+4x+5}\,dx$ S.M. or P.F.? (try S.M.

$u = 9x^2+4x+5$

$du = 2(9x+2)\,dx$ or $dx = \frac{1}{2}\left(\frac{1}{9x+2}\right)du$

$F = \int \frac{9x+2}{u}\cdot\frac{1}{2}\cdot\frac{1}{9x+2}\cdot dx = \frac{1}{2}\int\frac{1}{u}\,du$

$F = \frac{1}{2}\ln u + C$ or $\boxed{F = \ln\left\{\sqrt{9x^2+4x+5}\right\}+C}$

P1: (2) $F = \int \frac{9x+2}{x^2+x-6}\,dx$ P.F. $x^2+x-6 = (x+3)(x-2)$

note: $\frac{9x+2}{(x+3)(x-2)} = \frac{A}{x+3}+\frac{B}{x-2} = \frac{A(x-2)+B(x+3)}{(x+3)(x-2)}$

when: $x=-3$, $9(-3)+2 = -5A$ $\therefore A = 5$

and: $x=+2$, $9(2)+2 = 5B$ $\therefore B = 4$

$F = 5\int\frac{1}{x+3}\,dx + 4\int\frac{1}{x-2}\,dx$

$F = 5\ln(x+3) + 4\ln(x-2) + C$

or $\boxed{F = \ln\left\{(x+3)^5\cdot(x-2)^4\right\}+C}$

P2: (3) $F = \int t e^{-\frac{1}{2}t}\, dt$ I.P. $u = t$, $du = dt$

integrate $dv = e^{-\frac{1}{2}t} \cdot dt$ → $v = -2e^{-\frac{1}{2}t}$

$F = t(-2e^{-\frac{1}{2}t}) - \int (-2e^{-\frac{1}{2}t})\, dt$

$F = -2te^{-\frac{1}{2}t} + 2\left(-2e^{-\frac{1}{2}t}\right) + C$

$\therefore \boxed{F = -2(t+2)e^{-\frac{1}{2}t} + C}$

P2: (4) $F = \int \frac{dt}{5t^2+1} = \frac{1}{\sqrt{5}} \int \frac{1}{x^2+1}\, dx$

S.M. $x = \sqrt{5}\,t$ $(x^2 = 5t^2)$

$\boxed{F = \frac{1}{\sqrt{5}} \arctan(x) + C}$

P3: (5) $F = \int \sin(x)\cos^5(x)\, dx = \int z^5\, dz = \frac{1}{6}z^6 + C$

$\boxed{F = \frac{1}{6}\cos^6(x) + C}$

P3: (6) $F = \int x \cdot \arctan(x)\, dx = \frac{1}{2}x^2 \cdot \arctan(x) - \frac{1}{2}\int \frac{x^2}{1+x^2}\, dx$

I.P. $u = \arctan(x)$, $du = \frac{1}{1+x^2}\, dx$

$dv = x \cdot dx$

$v = \frac{1}{2}x^2$

← polynomial division

But $\int \frac{x^2}{1+x^2}\, dx = \int 1\, dx - \int \frac{1}{1+x^2}\, dx$

$\therefore \boxed{F = \frac{1}{2}(x^2+1)\arctan(x) - \frac{1}{2}x + C}$

P4: (7) $F = \int (\theta+1)^2 d\theta = \int z^2 dz = \frac{1}{3} z^3 + C$

$$\boxed{F = \frac{1}{3}(\theta+1)^3 + C}$$

S.M. $z = \theta + 1$

P4: (8) $F = \int \ln(w^2)\, dw = 2\int \ln(w)\, dw$

$$\boxed{F = 2w\left(\ln(w) - 1\right) + C}$$

E.C.E. cheat sheet

P5: (9) $F = \int \sqrt{t}\, e^{t\sqrt{t}}\, dt$

S.M. $u = t^{3/2}$

$du = \frac{3}{2} t^{1/2} dt$

$F = \frac{2}{3}\int e^u du = \frac{3}{2} e^u + C$

$$\boxed{F = \frac{2}{3} e^{t\sqrt{t}} + C}$$

P5: (10) $F = \int \frac{y\, dy}{(y^2-1)^{3/2}}$

S.M. (or T.S.)

$z = y^2 - 1$

$dz = 2y \cdot dy$

$dy = \frac{1}{2y} dz$

$F = \frac{1}{2}\int z^{-3/2} dz$

$F = \frac{1}{2} \cdot 2 \cdot z^{-1/2} + C$

$$\boxed{F = \frac{1}{\sqrt{y^2-1}} + C}$$