Object 1 is orbiting object 2. Approximate how the position and velocity of object 1 evolve after 0.1 seconds. (At time t = 0.1 seconds)

\[ r_x(0) = 0.5 \text{ m} \quad r_y(0) = 0 \text{ m} \]
\[ v_x(0) = 0 \text{ m/s} \quad v_y(0) = 1.5 \text{ m/s} \]

\[ F = ma \quad \text{or} \quad F = m \frac{dv}{dt} \quad \text{F = gravitational force on 1 from 2.} \]

\[ \Rightarrow F = -G \frac{Mm}{r^2} \hat{r} = -G \frac{Mm}{r^2} \frac{d}{dt} r \]
\[ \Rightarrow G \frac{Mm}{r^2} \hat{r} = m \frac{d^2}{dt^2} r \quad \text{(to keep calculation simple \( \cos = 1 \))} \]
\[ \Rightarrow \frac{d^2 r}{dt^2} = \frac{1}{r^2} \frac{d}{dt} r \]
break into components \( x \) \& \( y \)

\[ \frac{r_x}{r} = \frac{d}{dt} r_x \]
\[ \frac{r_y}{r} = \frac{d}{dt} r_y \]
where \( r = |r| = \sqrt{r_x^2 + r_y^2} \)

\[ t = 0.1 \text{ sec} \]
\[ \text{Tip: Start with velocity values} \]

\[ v_x(0.1) = v_x(0) + \Delta t \frac{dv_x}{dt} = v_x(0) + \Delta t \frac{d}{dt} \]
\[ r_x(t) = \frac{r_x(t)}{r_x(0)} = 0.5 \text{ m} \]
\[ \Rightarrow v_x(0.1) = 0.5 + 0.1s \left( \frac{0.5}{0.5} \right) 0.5^2 = 0.9 \text{ m/s} \]

\[ r_x(0.1) = r_x(0) + \Delta t \frac{dr_x}{dt} = r_x(0) + \Delta t v_x(0) \]
Choose \( V_x(0) \)? \( V_x(0.1) \)? or \( V_x(\text{some point}) \)?

Usually an average is better. Why?

\[ r_x \]

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\[ V_x(0) \]

**Actual Orbital Path (for example)**

**Approximation projects into the future**

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**Projected Value**

**Actual Orbital Path**

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**Side Note:**

We would like to be able to do this in choosing \( V_x(0.1) \) but we don’t know anything about the future for \( \frac{dx}{dt} \) so we are stuck using \( \frac{dV_x(0)}{dt} \)

So, let's use the mean velocity value \( \frac{V_x(0) + V_x(0.1)}{2} \)

\[ r_x(0.1) = 0.5 \text{ m} + 0.1 \text{ (0.2 m)} = 0.98 \text{ m} \]
\[ V_y(0) = V_y(0) + \Delta t \frac{dy}{dt} \]
\[ = 1.5 \% + (0.12) \left( \frac{9}{2} \right) \frac{\%}{2} = 1.5 \% \]
\[ r_y(0.1) = r_y(0) + \Delta t \left( \frac{V_y(0) + V_y(0.1)}{2} \right) \]
\[ = 0 \text{ m} + (0.12)(1.5 \%) = 0.15 \text{ m} \]

This method is only valid for \( m \ll M \)

Here \[ \frac{dV}{dt} = -G \frac{M}{r^2} \]

If we use a true inertial reference frame (non-accelerating)

Then \[ \frac{dV}{dt} = -G \frac{M + m}{r^2} \]

Remember our coordinate frame had the origin at object '2', which is not inertial.

\[ \Rightarrow \] Technically both objects are orbiting, however if one object is much larger than the other it is as if the smaller object is only orbiting the larger object (bigger object is inertial).