A few words on 'chaos':

- Sensitive dependence on initial conditions.
  * We saw this with the small cube of particles in the Lorenz vector field
  * Any small errors will be amplified (exponentially) in time

- Example: Double Pendulum Demo.
  * Very simple physical system that exhibits chaos
    * Four dimensional phase space: \(\frac{d}{dt} \theta_1 = \ldots\)
      trying to fit into
      \(\frac{d}{dt} \theta_2 = \ldots\)
      \(\frac{d}{dt} \theta_3 = \ldots\)
      \(\frac{d}{dt} \theta_4 = \ldots\)

- Chaos may be very challenging for numerical integration
  * Most integrators minimize local error at every time step
    However, even very small \((10^{-16})\) errors will grow rapidly in a chaotic system.

Best strategy
known
today.

Interestingly,
RK-78 is
approximately
symplectic...

* Alternative: instead of minimizing local errors, try to preserve conserved quantities:
  - Conserve energy \(\Rightarrow\) 'Symplectic' integrator
  - Make Lagrange's equations as close \(\Rightarrow\) 'Variational' integrator
to satisfied as possible
Example: predicting motion of the planets

* one of the oldest problems in physics/mathematics
* motivation for Poincare to discover chaos in first place.

Aside. \(\Rightarrow\) (also why Gauss discovered FFT in 1805),
(\textit{even two years before Fourier work})
(150 years before Cooley & Tukey).

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Example: Double Gyre ... model of ocean basin mixing,

* show demo Keynote
* show code.