Practice Problems for Midterm (Oct. 28, 2015)

Closed book. You may use one 8.5x11 sheet of handwritten notes (one side only).

1. Consider the cardioid \( r = 1 + \sin \theta \), \( 0 \leq \theta \leq 2\pi \), with parametric equations \( \theta(t) = t \), \( r(t) = 1 + \sin t \).
   (a) Write the position vector \( \vec{r}(t) \) of a point on this curve in terms of the polar basis functions.
   (b) Compute the velocity \( d\vec{r}/dt \) of a particle moving around the curve that reaches position \( \vec{r}(t) \) at time \( t \) in terms of the polar basis functions.
   (c) Write an integral for the arc length of the curve from \( t = 0 \) to \( t = 2\pi \). [You can evaluate this integral, but I probably wouldn’t ask you to do so on the midterm.]
   (d) Find the work done by a force field \( \vec{F} = r\hat{e}_r + \hat{e}_\theta \) on a particle moving along the curve from \( t = 0 \) to \( t = \pi/2 \).

2. Write the vector \( \vec{i} + \vec{j} + \vec{k} \) as a linear combination of the basis vectors \( \hat{e}_r, \hat{e}_\theta, \) and \( \hat{e}_z \) for cylindrical coordinates. Also write it as a linear combination of the basis vectors \( \hat{e}_r, \hat{e}_\theta, \) and \( \hat{e}_\phi \) for spherical coordinates. Also write it as a linear combination of the basis vectors from problem 3(b) on hw2.

3. Let a surface be represented by \( g(x, y, z) \equiv z - f(x, y) = 0 \). If \( \vec{r} \) is a vector representing a point on that surface, then \( \vec{r} = x\hat{i} + y\hat{j} + f(x, y)\hat{k} \). Show that \( \nabla g = (\partial\vec{r}/\partial x) \times (\partial\vec{r}/\partial y) \).

4. Find the surface area of the surface \( z = xy \), \( x^2 + y^2 \leq 1 \), \( x \geq 0 \), \( y \geq 0 \).

5. Verify Stokes’ Theorem when the surface \( S \) is the portion of the sphere \( x^2 + y^2 + z^2 = 1 \) in the first octant, and the boundary of \( S \) consists of the three circular arcs: \( x^2 + y^2 = 1 \), \( z = 0 \); \( y^2 + z^2 = 1 \), \( x = 0 \); and \( x^2 + z^2 = 1 \), \( y = 0 \). Assume that \( \vec{F} = x^2\hat{i} + 2xy\hat{j} + xz\hat{k} \).