Assignment 2.

Due Thursday, Apr. 12.

Reading: Chs. 2-3.

1. Exercise 4 of Chapter 2. Please note that there is a typo in the formula in part c: An exponent of $\frac{1}{4}$ is missing. It should say

$$\frac{dR}{dt} = -\left( \frac{a}{4M^{1/4}} \right) R.$$

2. Exercise 7 of Chapter 2.

3. Exercise 4 of Chapter 3.

4. In the 1999 movie *Office Space*, a character creates a program that takes fractions of cents that are truncated in a bank’s transactions and deposits them to his own account. This is not a new idea, and hackers who have actually attempted it have been arrested. In this exercise we will simulate the program to determine how long it would take to become a millionaire this way.

Assume that we have access to 50,000 bank accounts. Initially we can take the account balances to be uniformly distributed between, say, $100 and $100,000. The annual interest rate on the accounts is 5%, and interest is compounded daily and added to the accounts, except that fractions of a cent are truncated. These will be deposited to an illegal account that initially has balance $0.

(a) Take the following partial MATLAB code and fill in the indicated lines to simulate the *Office Space* scenario. [Alternatively, you may write a code in a language of your choice to do the same thing.]

```matlab
% Simulate "Office Space" scenario, where fractions of a penny from legal accounts are transferred to an illegal account. Determine how long before illegal account has % a million dollars.

accounts = 100 + (100000-100)*rand(50000,1); % Sets up 50,000 accounts with % balances between $100 and $100000.
accounts = floor(100*accounts)/100; % Deletes fractions of a cent from % initial balances.
illegal = 0; % illegal acct is initially 0.
days = 0;
```
while illegal < 10^6,
    % Continue until illegal acct % has a million dollars
    days = days + 1;
    accounts_new = % Add daily interest to accounts.
    %% Fill in formula for value of accounts after interest is added.
    accounts = floor(100*accounts_new)/100; % Delete fractions of a cent.
    illegal = % Add daily interest to illegal acct.
    %% Fill in formula for value of illegal acct after interest is added.
    illegal = illegal + % Also add fractions of a cent
    %% Fill in an expression for the total amount of money deleted from the other
    %% accounts that will now be added to illegal acct. (You may want to use
    %% the Matlab 'sum' command.)
    %
    end;
    days, illegal % Print results.

The first lines of the code set up the initial accounts. The MATLAB function `rand` generates uniformly distributed random numbers between 0 and 1. The function `floor` takes the largest integer that is less than or equal to its argument. Thus, if an account value is, say 527.125 dollars, we multiply by 100 to get 52712.5 cents, then take `floor` of this number to get 52712 cents. Finally to get the truncated amount back into dollars, we divide by 100 and have 527.12 dollars.

After setting up the accounts and initializing the illegal account to 0, we will now iterate until the amount in the illegal account reaches a million dollars ($10^6$), using a `while` loop. Inside that loop, the first thing you need to fill in is a formula for the value of the accounts after interest is added. (This is stored in the vector `accounts_new`). The line after that uses `floor` to delete fractions of a cent, just as was done in the initial setup. Next you need to fill in a formula for the value of the illegal account after interest is added. We will assume that this account can hold fractional amounts, so it will not be truncated. Finally, you need to add to the illegal account all of the money that was removed from the other accounts. (You may find the MATLAB `sum` command useful for this.)

Turn in a listing of your code or, at least, the lines that you filled in above as well as the answer to the question of how long does it take to become a millionaire this way.

(b) Without running your code, answer the following questions: On average about how much money would you expect to be added to the illegal account each day due to the embezzlement? Suppose you had access to 100,000 accounts, each initially with a balance of, say $5000. About how much money would be added
to the illegal account each day in this case? Explain your answers.