Semiconductor Detectors vs. Scintillator+PMT Detectors

- Semiconductors are emerging technology — Scint.PMT systems relatively unchanged in 50 years. NaI(Tl) excellent for single-photon, new scintillation materials being developed for PET (lutetium-based, GSO, LaBr₃, LaCl₃).
- Direct detection in semicon. permits superior energy resolution — Scint.PMT is indirect (2-step process) with marginal quantum efficiency, which limits energy resolution.
- Purity/growth of semicon. is still a challenge (i.e. expensive) — Scint.PMT is well established stable technology.
- Semicon. can be finely pixilated for spatial resolution (µm theoretically) — Scint.PMT pixel size cannot be made arbitrarily small (~ 0.5mm – 1.0mm lower limit thus far). Note spatial resolution / sensitivity trade-offs!
- Semicon. cannot be made arbitrarily thick for photon absorption efficiency (sensitivity) — Scintillation crystals can be arbitrarily thick. Note spatial resolution / sensitivity / energy resolution trade-offs!
- Semicon. require more sensitive electronics generally, and less amiable to signal multiplexing, resulting in potential need of many, many more processing channels.

Sample Spectroscopy System Hardware

(from: The Essential Physics of Medical Imaging (Bushberg, et al))

- NaI(Tl) crystal converted to ~20% visible photons
- High voltage power supply
- Pre-amplifier
- Multi-channel Analyzer
- Amplifier

Incoming high-energy gamma ray

Electron multiplication becomes electric signal

(higher gamma energy)

(statistical uncertainties!)

larger current or voltage
more electrons
more scintillation photons
Interaction Rate and Dead-time

From: The Essential Physics of Medical Imaging (Bushberg, et al)

Interactions of Photons with a Spectrometer

A. Photoelectric
B. Compton + Photoelectric
C. Compton
D. Photoelectric with characteristic x-ray escape
E. Compton scattered photon from lead shield
F. Characteristic x-ray from lead shield
Sample Spectroscopy System

Output

Ideal Energy Spectrum

counting mode

Energy Resolution

Realistic Energy Spectrum

From: Physics in Nuclear Medicine (Sorenson and Phelps)
Sample Spectrum (Cs-137)

A. Photopeak
B. Compton continuum
C. Compton edge
D. Backscatter peak
E. Barium x-ray photopeak
F. Lead x-rays

Sample Spectrum (Tc-99m)

A. Photopeak
B. Photoelectric with iodine K-shell x-ray escape
C. Absorption of lead x-rays from shield

Note absence of Compton continuum

Why?

From: The Essential Physics of Medical Imaging (Bushberg, et al)
Effects of Pulse Pileup

Fig. 11-10. (A) \(^{99m}\text{Tc}\) spectrum at low counting rate. (B) Spectral broadening and shift in apparent photopeak energy due to pulse pileup and baseline shift in the spectrometer amplifier at high counting rate.

Calibrations

- Energy calibration (imaging systems/spectroscopy)
  - Adjust energy windows around a known photopeak
  - Often done with long-lives isotopes for convenience
    - Cs-137: \(E_{\gamma} = 662\) keV (close to PET 511 keV), \(T_{1/2} = 30\) yr
    - Co-57: \(E_{\gamma} = 122\) keV (close to Tc99m 140 keV ), \(T_{1/2} = 272\) d

- Dose calibration (dose calibrator)
  - Measure activity of know reference samples (e.g., Cs-137 and Co-57)
  - Linearity measured by repeated measurements of a decaying source (e.g., Tc-99m)
Overview of today’s lecture

- Emission vs. Transmission Imaging
- Nature of nuclear radiation
  - Isotopes used in nuclear medicine
- Detection methods
- **Counting statistics**
- Imaging systems
  - Planar gamma scintigraphy

Random Processes in Nuclear Medicine

Radiation decay and detection are random processes described by probabilistic statistical distribution functions; $P(x, t, \ldots)$.

The means and variances of these probability distributions are part of the data analysis process in nuclear medicine.

Example governed by quantitative law:

Radioactive decay in time;

$$P(t)dt = \exp\left(-\frac{t}{\tau}\right)dt$$

probability atom will decay between the times $t$ and $t+dt$. [$\tau = T_{1/2} / \ln(2)$, $T_{1/2}$ = half-life of isotope]

Gamma ray absorption in medium;

$$P(x)dx = \exp(-\mu x)dx$$

probability $\gamma$ will be absorbed between $x$ and $x+dx$. $\mu$ = attenuation coefficient of material at specified energy.

Examples governed by empirical law:

Scintillation photons created upon absorption of a gamma ray;

There is a mean number created per unit of absorbed energy, and an associated variance, for each scintillation material. Have to work with law of averages $\rightarrow$ probabilities.

Scintillation photon creates a photo-electron at a PMT photocathode;

Photocathodes have an intrinsic quantum efficiency (QE) — the chance that any given photon creates a photo-electron is QE%. After observing 1000s of photons, QE% will have generated photo-electrons.
Characterizing Random Phenomena (and Errors)

Measures of Central Tendency:
• **Mode** – Most Frequent Measurements (not necessarily unique)
• **Median** – Central Value dividing data set into 2 equal parts (unique term)
• **Mean** (Arithmetic Mean) \( \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \)

Measures of Dispersion:
• **Range** – Difference of largest and smallest values
• **Variance** – Measures dispersion around mean: \( \sigma^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \)
• **Standard Deviation**: \( \sigma = \sqrt{\sigma^2} \)

Statistical Models for Random Trials

• **Binomial Distribution**
  – Random independent processes with two possible outcomes

• **Poisson Distribution**
  – Simplification of binomial distribution with certain constraints

• **Gaussian or Normal Distribution**
  – Further simplification if average number of successes is large (e.g., >20)
Characterizing Random Errors With a Distribution

**Binomial Distribution** - Independent trials with two possible outcomes

Binomial Density Function: 

$$P_{\text{binomial}}(r) = \frac{n!}{r!(n-r)!} p^r (1-p)^{n-r}$$

Probability of $r$ successes in $n$ tries when $p$ is probability of success in single trial

$$\bar{x} = np \quad \sigma = \sqrt{np(1-p)}$$

**Example:** What is the probability of rolling a 1 on a six sided die exactly 10 times when the die is rolled for a total of 24 times.

$r = 10$, $n = 24$, $p = 1/6$, $P_{\text{binom}}(r=10) = 0.0025 \sim 1$ in 400

**Poisson Distribution** - Limiting form of binomial distribution as $p \to 0$ and $n \to \infty$

As in nuclear decay. Have many, many nuclei, probability of decay and observation of decay very, very small

$$P_{\text{Poisson}}(r) = \frac{\mu^r \exp(-\mu)}{r!}$$

Only one parameter: $\mu = \text{Mean} = \text{Variance} \implies \bar{x} = \sigma^2$

**Example:** A radioactive source is found to have a count rate of 5 counts/second. What is probability of observing no counts in a period of 2 seconds?

$$P_{\text{Poisson}}(r=0) = \frac{(\mu=10)^0 \exp(-\mu)}{(r=0)!} = 4.54 \times 10^{-5}$$

**Gaussian (Normal) Distribution**

- Symmetric about the mean
- Useful in counting statistics because distributions are **approximately** normal when $n \gg 20$
- Variance and mean not necessarily equal

$$P_{\text{Gaussian}}(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{(x-\mu)^2}{2\sigma^2} \right)$$

![Gaussian distribution graph](image)
Variance / Error in Counting Photons

Poisson process: mean = variance

—-> Number measured, $N$, is best estimate of mean number for that phenomenon (e.g. $N$ emitted gamma rays per unit time, $N$ scintillation photons per absorbed gamma ray, …)

—-> variance = mean = $N$

—-> standard deviation: $\sigma = \sqrt{\text{variance}} = \sqrt{N}$

Relative error, $e$, in counting experiments:

$e = \frac{\sigma}{\bar{x}} = \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}}$

Relative error decreases as number of events increases

Emphasizes the importance of detecting as many gamma rays as possible, and the sensitivity (absorption efficiency) of nuclear medicine cameras

- This applies to individual image pixels in nuclear medicine (also applies to x-ray imaging, but number of photons is not limited there)
- Also applies to energy resolution in radiation detection systems

Simple Propagation of Error

- Quantities of interest are often determined from several measurements prone to random error.
- If the quantities are independent, then add independent contributions to error in quadrature as follows:

The simplest examples are addition, subtraction, and multiplication by a constant.

If the quantities $a$ and $b$ are measured with known error $\delta_a$ and $\delta_b$, then the error in the quantities $x$, $y$, $z$ when

$x = a + b$
$y = a - b$
$z = k*a$, $k$ = constant (no error)

are:

$\delta_x = \delta_y = \sqrt{\delta_a^2 + \delta_b^2}$
$\delta_z = k \delta_a$
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The Planar Gamma Camera