1. (10) Crude oil having kinematic viscosity of $7.5 \times 10^{-5}$ m$^2$/s and density of 880 kg/m$^3$ is pumped through a pipe of 0.75 m diameter at an average velocity of 1.2 m/s. The roughness of the pipe is equivalent to that of a “commercial steel” pipe. If pumping stations are 330 km apart, find the head loss (in meters of oil) between pumping stations and the power required to overcome friction.

**Solution:**

Commercial steel pipe:

$$k_s = 0.046 \text{ mm} \quad \text{so} \quad k_s \frac{D}{750 \text{ mm}} = 6.1 \cdot 10^{-5}$$

$$Re = \frac{VD}{\nu} = \frac{1.2 \text{ m/s} \cdot 0.75 \text{ m}}{7.5 \cdot 10^{-5} \text{ m}^2/\text{s}} = 12000$$

Figure 5-4 the gives us $f = 0.029$. The head loss due to friction is then

$$h_f = f \frac{L V^2}{D 2g} = 0.029 \cdot \frac{330000 \text{ m}}{0.75 \text{ m}} \cdot \frac{1.2 \text{ m/s}^2}{2 \cdot 9.81 \text{ m/s}^2} = 937 \text{ m}.$$

$$P_f = Q \gamma h_f = V \frac{\pi D^2}{4} \rho gh_f = 1.2 \text{ m/s} \cdot \frac{\pi (0.75 \text{ m})^2}{4} \cdot 880 \text{ kg/m}^3 \cdot 9.81 \text{ m/s}^2 \cdot 937 \text{ m}$$

$$= 4.29 \cdot 10^6 \text{ W} = 4.29 \text{ MW}.$$

2. (10) What discharge will produce cavitation (i.e. vapor pressure) in the Venturi tube for the configuration shown if the indicated dimensions are $d_1 = 2.5$ cm, $d_2 = 10$ cm, $d_3 = 5$ cm and $L = 310$ m? The friction coefficient for the pipe of length $L$ is $f = 0.04$, and the head loss due to flow expansions of the Venturi tube is $0.1V_1^2/2g$. Assume that the fluid is water at 20°C.

**Solution:**

The vapor pressure $p_v = 2.34$ kPa and atmospheric pressure $p_a = 101$ kPa. Specific weight of the water is $\gamma = 9790$ N. Write the energy equation from point 1 to point 3:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$
where

\[ h_L = 0.1 \frac{V_1^2}{2g} + f \frac{L V_2^2}{2d_2^2}. \]

We have

\[ V_i = \frac{Q}{A_i} = \frac{4Q}{\pi d_i^2} \]

so the energy equation can be rewritten as

\[ \frac{p_1}{\gamma} + \frac{8Q^2}{\pi^2 g d_1^4} = \frac{p_3}{\gamma} + \frac{8Q^2}{\pi^2 g d_3^4} + \left( 0.1 \frac{1}{d_1^4} + f \frac{310 \text{ m}}{d_2^5} \right) \frac{8Q^2}{\pi^2 g} \]

giving

\[ \frac{8Q^2}{\pi^2 g} \left( \frac{1}{d_1^4} - \frac{1}{d_3^4} - 0.1 \frac{1}{d_1^4} - f \frac{L}{d_2^5} \right) = \frac{p_3 - p_1}{\gamma} \]

and setting \( p_1 = p_e \) and \( p_3 = p_a \) gives us then

\[ Q^2 = \frac{\pi^2 g (p_a - p_e)}{8\gamma \left( \frac{0.9}{d_1^4} - \frac{1}{d_3^4} - f \frac{L}{d_2^5} \right)} \]

\[ = \frac{\pi^2 \cdot 9.81 \text{ m/s}^2 \cdot (1.01 \cdot 10^5 \text{ N/m}^2 - 2340 \text{ N/m}^2)}{8 \cdot 9790 \text{ N/m}^3 \cdot \left( \frac{0.9}{(0.025 \text{ m})^4} - \frac{1}{(0.05 \text{ m})^4} - 0.04 \frac{310 \text{ m}}{(0.1 \text{ m})^5} \right)} \]

\[ = 1.35 \cdot 10^{-4} \text{ m}^6/\text{s}^2. \]

So

\[ Q = 0.0116 \text{ m}^3/\text{s} \quad (= 11.6 \text{ l/s}). \]