1. (18) Consider the condition that the initial flow velocity in the system shown below is 1.8 m/s and the pipe is a 0.3 m in diameter, 6 mm thick, 2000 m long steel pipe. If the valve at the downstream end is closed instantaneously at $t = 0$, and neglecting both friction and minor (local) losses:

a) What does the pressure head vs. time trace look like at point $A$ for the next 4.0 seconds, indicating the values of maximum and minimum pressure heads and the time.

b) Plot the pressure head vs. position along the pipe at $t = 2.0$ s, providing all relevant values (i.e. the values of pressure and the location of pressure change, etc.).

c) Plot the flow (magnitude and direction) vs. position in the pipe at $t = 2.0$ s, providing all relevant values.

d) Plot a characteristic line in the $x$-$t$ plane emanating from the valve position from $t = 0$ to 2.0 s.

**Solution:**

\[
a = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{2.2 \times 10^6 \text{ Pa}}{998 \text{ kg/m}^3}} = 1200 \text{ m/s}.
\]

\[
\Delta H = -\frac{a}{g} \Delta V = \frac{1200 \text{ m/s}}{9.81 \text{ m/s}^2} \cdot 1.8 \text{ m/s} = 220 \text{ m}.
\]

a) Travel time of pressure wave from valve to point $A$:

\[
t_1 = \frac{1400 \text{ m}}{1200 \text{ m/s}} = 1.17 \text{ s}.
\]

From point $A$ to the reservoir and back:

\[
t_2 = t_1 + \frac{2 \cdot 600 \text{ m}}{1200 \text{ m/s}} = 1.17 \text{ s} + 1.00 \text{ s} = 2.17 \text{ s}.
\]

From point $A$ to the valve and back:

\[
t_3 = t_2 + \frac{2 \cdot 1400 \text{ m}}{1200 \text{ m/s}} = 2.17 \text{ s} + 2.33 \text{ s} = 4.50 \text{ s}.
\]
Time history of pressure head at point A:

b) Position of wave front at $t = 2.0$ s:

\[ x = at = 1200 \text{ m} \cdot 2.0 \text{ s} = 2400 \text{ m} = L + 400 \text{ m} \]

i.e. the wave has travelled the full length of the pipe and 400 m back.
c) Velocity at $t = 2.0$ s:

\[ V[m/s] \]

\[ x[m] \]

\[ 0 \]

\[ 400 \]

\[ A \]

\[ 0 \]

\[ 1.80 \]

\[ 0 \]

\[ x[m] \]

\[ 0 \]

\[ 400 \]

\[ A \]

\[ 0 \]

\[ 0 \]

\[ 0 \]

\[ t[s] \]

\[ 0 \]

\[ 2.00 \]

\[ x[m] \]

\[ 0 \]

\[ 400 \]

\[ A \]

\[ 0 \]

\[ 0 \]

\[ t[s] \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ x[m] \]

\[ 0 \]

\[ 400 \]

\[ A \]

\[ 0 \]

\[ 0 \]

\[ t[s] \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ x[m] \]

\[ 0 \]

\[ 400 \]

\[ A \]

\[ 0 \]

\[ 0 \]

\[ t[s] \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ x[m] \]

\[ 0 \]

\[ 400 \]

\[ A \]

\[ 0 \]

\[ 0 \]

\[ t[s] \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ x[m] \]

\[ 0 \]

\[ 400 \]

\[ A \]

\[ 0 \]

\[ 0 \]

\[ t[s] \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ x[m] \]

\[ 0 \]

\[ 400 \]

\[ A \]

\[ 0 \]

\[ 0 \]

\[ t[s] \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ x[m] \]

\[ 0 \]

\[ 400 \]

\[ A \]

\[ 0 \]

\[ 0 \]

\[ t[s] \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ x[m] \]

\[ 0 \]

\[ 400 \]

\[ A \]

\[ 0 \]

\[ 0 \]

\[ t[s] \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ x[m] \]

\[ 0 \]

\[ 400 \]

\[ A \]

\[ 0 \]

\[ 0 \]

\[ t[s] \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ x[m] \]

\[ 0 \]

\[ 400 \]

\[ A \]

\[ 0 \]

\[ 0 \]

\[ t[s] \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ x[m] \]

\[ 0 \]

\[ 400 \]

\[ A \]

\[ 0 \]

\[ 0 \]

\[ t[s] \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ x[m] \]

\[ 0 \]

\[ 400 \]

\[ A \]

\[ 0 \]

\[ 0 \]

\[ t[s] \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ x[m] \]

\[ 0 \]

\[ 400 \]

\[ A \]

\[ 0 \]

\[ 0 \]

\[ t[s] \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ x[m] \]

\[ 0 \]

\[ 400 \]

\[ A \]

\[ 0 \]

\[ 0 \]

\[ t[s] \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ 0 \]

\[ 2.00 \]

\[ 1.67 \]

\[ x[m] \]

\[ 0 \]

\[ 400 \]

\[ A \]

\[ 0 \]

\[ 0 \]

\[ t[s] \]

\[ 0 \]
2. (7) The same situation as Problem 1, but the valve is initially closed, hence the flow in the pipe is quiescent. If the valve is opened instantaneously at \( t = 0 \), how long does it take for the flow to reach 1.0 m/s.

**Solution:**

\[
\frac{V}{V_0} = \frac{1.0 \text{ m/s}}{1.8 \text{ m/s}} = 0.556
\]

\[
t = \frac{LV_0}{2gH_0} \ln \left[ \frac{1 + \frac{V}{V_0}}{1 - \frac{V}{V_0}} \right] = \frac{2000 \text{ m} \cdot 1.8 \text{ m/s}}{2 \cdot 9.81 \text{ m/s}^2 \cdot 30 \text{ m}} \ln \left[ \frac{1.556}{0.444} \right] = 7.66 \text{ s}.
\]