Modified Berggren Equation

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Stefan Formula

- Assumes latent heat of the soil is the only heat which must be removed when freezing the soil.
- This is an over simplification of the actual conditions.
Stefan Formula
Stefan Formula

Heat removal process can be represented by

\[ Q_1 = L \frac{dx}{dt} \rightarrow \text{heat released by freezing a layer of soil } dx \text{ thick in time } dt \]

\[ Q_2 = \frac{\Delta T}{R} = \frac{\Delta T}{(x/k_f)} \rightarrow \text{heat conducted through frozen layer} \]

and

\[ Q_1 = Q_2 \Rightarrow L \frac{dx}{dt} = k_f \frac{\Delta T}{x} \]

by integrating and solving for \( x \).

\[ x = \sqrt{\frac{2k_f}{L}} \int \Delta T \, dt \]

(Eq. 2.1)
Stefan Formula

• Refer to Para 3.2.2.2 and Equation 2.1

\[ \int (\Delta T) (dt) = \text{surface freezing index (°F-hr).} \]

Freezing index normally expressed as °F-days. Thus expression is \((2)(24)(n)(FI) = 48nFI\)
Modified Berggren Equation

• Best reference for Assignment No. 3 is PGI (1995), Volume 2, Section 2.0, Paragraph 3.2.2

• Modified Berggren Equation:

\[ x = \lambda \sqrt[48]{48(k_{avg})(n)(FI)} \]
Inputs

• $k = \text{thermal conductivity} = \text{BTU/hr-ft}^2\cdot{}^\circ\text{F/ft}$
  
  $= \text{BTU/hr-ft}^{-{}\circ}\text{F}$

• $k$ for pavement material's $f(\text{density, mc})$

• $k$ for HMA?
**Inputs**

\( k_{\text{avg}} \)

- Average thermal conductivity of each layer. First, you must estimate the layer moisture content and dry density (see Para 3.2). If you want to use figures for determination of \( k \), refer to Figs 2.28-2.30. Get \( k \) values for frozen and unfrozen cases, then average.
Inputs

Sources for equation inputs:

• **FI**: Average FI is given for most Washington State cities in Table 2.10. Use the contour map (Fig 2.33) for Design FI. Units are °F-days.

• **n**: Adjusts air FI (which is what you get from Table 2.10 or Fig 2.33) to surface FI. Refer to “Typical Values” in Para 3.2.1.5(c)(i).

\[ n = \frac{\text{surface freezing index}}{\text{air freezing index}} \]
Inputs

Figure 2.33. Design Annual Freezing Index Contour Map
Inputs

Sources for equation inputs:

- **L**: Latent heat is the heat that must be removed to convert an unfrozen soil to frozen at 32°F. Function of the layer density and moisture content (refer to Para 3.2.1.4).

\[
L = (144 \text{ BTU/lb})(w)(\gamma_d) = \text{BTU/ft}^3
\]
Modified Berggren Equation

• Must deal with multiple layers for most conditions (and Assignment 3)
• Use the following

\[
x = \lambda \sqrt{\frac{48(n)(FI)}{(L/k)_{\text{eff}}}}
\]
Use of Modified Berggren Equation

• How do you calculate \((L/k)_{\text{eff}}\)?
• First, **assume** a depth of freeze, \(x\)
• Second, use

\[
(L/k)_{\text{eff}} = \frac{2}{x^2} \left[ \frac{z_1}{k_1} \left( \frac{L_1 z_1}{2} + L_2 z_2 + L_3 z_3 \right) + \frac{z_2}{k_2} \left( \frac{L_2 z_2}{2} + L_3 z_3 \right) + \frac{z_3}{k_3} \left( \frac{L_3 z_3}{2} \right) \right]
\]

This is the assumed total depth of freeze (ft)