1. Briefly explain how an RTMS works and why it can detect motionless vehicles [6 points].

A Remote Traffic Microwave Sensor employs the Frequency Modulated Continuous Waveform (FMCW) principle. The transmitted signal frequency continuously varies over time. A RTMS uses the frequency change in the time period between emission and reception of the modulating signal to calculate the range to the object.

A RTMS can detect motionless vehicles because the range will be different when a vehicle is present and reflects the signal back to the RTMS. If the footprint of a RTMS is set perpendicular to the roadway, it can count volume and occupancy for up to eight lanes. When instead the footprint is placed parallel over roadway, it can be used on a per lane basis to set up bins for speed measurements.

2. You are asked to choose an RTMS for a traffic detection work. The minimum resolution required is 3.5 meters. Two RTMS products, product A and product B, are available. Product A operates in the K band and product B operates in the X band. The radio frequency (RF) modulation bandwidth for product A is 45 MHz, and that for product B is 40 MHz. Which product should you choose? Why? [6 points]

The range resolution ($\Delta R$) can be calculated as follows:

$$\Delta R = \frac{c}{2\Delta F}$$

where

- $\Delta F$ = radio frequency modulation bandwidth; and
- $c$ = constant (speed of light)

For product A:

$$\Delta R_A = \frac{c}{2\Delta F} = \frac{3 \times 10^8}{2 \times 45 \times 10^6} = 3.33m < 3.50m$$

For product B:

$$\Delta R_B = \frac{c}{2\Delta F} = \frac{3 \times 10^8}{2 \times 40 \times 10^6} = 3.75m > 3.50m$$

Thus, product A meets the requirement but product B does not. Product A should be chosen for the detection work.

3. According to the ferromagnetic effect, the inductance of a loop detector should increase when a vehicle drives over it. However, we observe a decrease of inductance when a vehicle is on the loop. What is the reason? [6 points]
The iron mass of the vehicle engine, transmission or differential will increase the inductance of the loop detector due to the ferromagnetic effect. However, the eddy currents resulting from the metal on the vehicle’s periphery will reduce the inductance of the loop detector. This reduction due to the eddy currents is greater than the increase caused by the iron mass. Therefore, there is an overall reduction in loop inductance when a vehicle passes over the loop.

4. Single loop measurements can be used to estimate space mean speed. Typically, speed is calculated as follows:

\[
\tilde{s}(i) = \frac{N(i)}{T \cdot O(i) \cdot g}
\]

(1)

Where 
\(i\) = time interval index, 
\(\tilde{s}\) = space-mean speed; 
\(N\) = volume (vehicles per interval), 
\(O\) = lane occupancy, 
\(T\) = time length of a measurement interval, and 
\(g\) = speed estimation parameter.

(1) What is the relationship between \(g\) and the mean effective vehicle length? (show how the relationship is developed).
(2) Under what circumstances, can \(g\) be regarded as a constant?
(3) If \(g\) is chosen to be a constant of 2.4, what are the possible effects on estimated speeds? [12 points, 4 points for each question]

(1)

The relationship between \(g\) and the mean effective vehicle length, \(\bar{l}(i)\), can be expressed using equation (4-1) as follows:

\[
g = \frac{1}{\bar{l}(i)}
\]

(4-1)

Where
\(i\)=index of the interval.

Equation (4-1) can be derived as follows:

For an interval \(i\), the occupancy \(O(i)\) can be calculated using equation (4-2):

\[
O(i) = \frac{1}{T} \sum_{j=1}^{N(i)} \frac{l_j(i)}{S_j(i)}
\]

(4-2)

Where
\(T\)= interval length (20 second for WSDOT system, 30 for CalTrans system); 
\(l_j(i)\) = length of the \(j^{th}\) vehicle in the \(i^{th}\) interval; and 
\(S_j(i)\) = speed of the \(j^{th}\) vehicle.
If we assume that during this interval the speed is equal for all the \( N(i) \) vehicles that have passed by, Equation (4-2) can be rearranged to get Equation (4-3):

\[
O(i) = \frac{1}{T \cdot S(i)} \sum_{j=1}^{N(i)} l_j(i) \quad (4-3)
\]

Rearrange Equation (4-3) so that \( S(i) \) is on the left side of the equation, we get Equation (4-4):

\[
S(i) = \frac{1}{T \cdot O(i)} \sum_{j=1}^{N(i)} l_j(i) \quad (4-4)
\]

Let’s \( \bar{l}(i) \) denotes average equivalent vehicle length for the \( i \)th interval. Since we know that \( \sum_{j=1}^{N(i)} l_j(i) = N(i) \cdot \bar{l}(i) \), Equation (4-4) can be expressed as follows:

\[
\bar{S}(i) = \frac{N(i) \cdot \bar{l}(i)}{T \cdot O(i)} \quad (4-5)
\]

Comparing Equation (1) with Equation (4-5), we get \( g = \frac{1}{\bar{l}(i)} \). More accurately, \( g(i) = \frac{1}{\bar{l}(i)} \). The relationship between \( g \) and mean effective vehicle length has been derived.

(2)

\( g \) can be regarded as a constant only when the mean (effective) vehicle lengths are approximately identical from interval to interval.

(3)

If \( g = 2.4 \), the corresponding mean effective vehicle length is 22 ft. When the mean effective vehicle length of an interval is shorter than 22 ft, the mean speed of the interval will be overestimated. If the mean effective vehicle length of an interval is longer than 22 ft, the mean speed of the interval will be underestimated.

5. The inductance of an inductive loop detector decreases when a metal vehicle passes over the loop. The decreased inductance increases the oscillation frequency of the electronics circuit of which the inductive loop is a part. The oscillatory frequency, \( f \), of the circuit is given by

\[
f = \frac{1}{2\pi \sqrt{L_D C_D}} \quad (2)
\]

when the quality factor \( Q > 5 \). The factor \( L_D \) is the total inductance across the inductive loop terminals and \( C_D \) is the sum of the total capacitance across the inductive loop terminals and the internal tuning capacitance of the electronics circuit. The ratio of the
change in oscillatory frequency produced by a vehicle to the frequency in the absence of a vehicle is given by

\[
\frac{\Delta f}{f} = -\frac{\Delta L_D}{2L_D}
\]  

(3)

(1) Derive Equation (3) from Equation (2).

(2) If \( f \) is 20 KHz and \( \Delta L_D/L_D \) (known as the sensitivity of the detector to an inductance change) is -3 percent, what is the change in frequency sensed by the electronic circuit? If \( \Delta f \) is measured to be 600 Hz, what is the sensitivity (\( \Delta L_D/L_D \))?

[10 points, 5 point for each question]

(1) Take first derivative of \( f \) on \( L_D \) on both sides of Equation (2), we can get the following equation (5-1):

\[
\frac{\Delta f}{\Delta L_D} = -\frac{1}{2\pi \sqrt{C_D}} \left( -\frac{1}{2 \sqrt{L_D}} \right) = -\frac{1}{2\pi \sqrt{L_D} C_D} \left( -\frac{1}{2 L_D} \right)
\]

(5-1)

Substitute Equation (2) to Equation (5-1), we get the following equation:

\[
\frac{\Delta f}{\Delta L_D} = f \left( -\frac{1}{2 L_D} \right)
\]

(5-2)

Move \( f \) to the left-side of the equation, and move \( \Delta L_D \) to the right side of the equation, we get the following equation:

\[
\frac{\Delta f}{f} = -\frac{\Delta L_D}{2L_D}
\]

(3)

Thus, Equation (3) has been derived from Equation (2).

(2)

Given \( \Delta L_D/L_D = -3\% \) and \( f = 20 \) KHz. According to Equation (3), \( \Delta f \) can be calculated as follows:

\[
\Delta f = -\frac{1}{2} \times (-3\%) \times 20 \times 10^3 \text{Hz} = 300 \text{Hz}
\]

The change in frequency sensed by electronic circuit is 300 Hz.

If \( \Delta f = 600 \text{Hz} \), \( \Delta L_D/L_D = -2 \times 600 \text{Hz} / 20 \times 10^3 \text{Hz} = -6\%

The sensitivity (\( \Delta L_D/L_D \)) is -6 percent.