A mathematical model is described which relates collection efficiency to precipitator size and operating parameters. Procedures are given for calculating particle charging rates, electric field as a function of position in wire-plate geometry, and the theoretically expected collection efficiencies for various particle sizes and precipitator operating conditions. Methods are proposed for empirically representing the losses in collection efficiency caused by non-uniform gas velocity distributions, gas by-passage of the electrostatic region, and particle reentrainment due to rapping of the collection electrode. Incorporation of these proposed techniques into a mathematical model of precipitator performance results in reduction of the theoretically calculated overall collection efficiencies. The reduced efficiencies are compared with those obtained from measurements on precipitators treating raw gas from qualified generating stations. The effects of changes in particle size distributions and precipitated collection efficiencies obtained from the semi-empirical model are also presented.

This paper summarizes work conducted at Southern Research Institute concerning the development of a mathematical model for simulating electrostatic precipitator performance. The approach which has been taken is to define the collection efficiency under ideal conditions in terms of precipitator geometry and operating parameters for diverse raw gas sizes and properties. Empirical corrections to the theoretical performance are then made to account for non-uniformity of gas flow and reentrainment of dust laden gas through non-isolated grid regions above and below the collection electrode.

The incentive for using a theoretical approach lies in the desire to account for the variations in such characteristics as precipitator operating parameters in a logical and orderly fashion. Methods which are exclusively empirical can lead to serious misestimation in collection electrode area requirements for a specific installation. A theoretical approach offers the potential for increased confidence in design and is one avenue by preventing under-design on the one hand and over-design on the other. The accuracy of predictions obtained from such an approach is subject to the accuracy with which the properties comprising the independent variables are measured, the degree to which the theoretical relationships describe precipitator operation, and the precision with which the factors that correct for non-ideal conditions can be modeled and determined. At present, it is necessary to use assumed values for parameters describing non-ideal conditions. Calculations between predicted and measured performance using the relationships described in the paper and the limited amount of applicable test data indicate that the model in its present stage of development is useful in developing performance curves for collection efficiency in specific collecting area under various conditions.

**Theoretical Background**

The fundamental steps in the precipitation process are particle charging, particle collection, and the removal and disposal of the collected material. Particle charging is accomplished by a source of charge carriers in the presence of an electric field which drives the charge carriers to the precipitator. Collection of the charged particle occurs as the electric field drives the particle to a collecting electrode, where they are held by mechanical and electrical forces. The removal of the collected material is accomplished by the application of a force to the collecting electrode in such a manner that the collected ash is digested and falls into a receiving hopper for subsequent transport to a disposal area.

In order to calculate the theoretically expected performance of an electrostatic precipitator, it is necessary to calculate particle charge as a function of particle size, residence time, and precipitator operating conditions. Field charging theory adequately describes experimentally observed particle charge values for particles larger than 2.0 um with moderate to high values of applied electric field. Calculation of particle charge for diameters exceeding 2.0 um is therefore relatively straightforward if the precipitator operating conditions are adequately defined. For particle diameters less than 2.0 um, the calculations of particle charge at all states, and has been the subject of considerable effort by a number of investigators. A more detailed discussion of particle charging theories is given in a paper by Smith and McDonald.

The expression currently used in the model for particle charging calculations is given below in differential form:

\[ \frac{dQ}{dt} = \frac{d}{dV(t)} \left[ 1 + \frac{Q}{V(t)} \right] \]

\[ \phi = \text{modified} \cdot \text{expression} \text{ charged} \]

where: \( \phi \) is a modified charge. It can be rewritten as:

\[ \phi = 4\pi \rho + \frac{1}{r} \left( 1 + \frac{2K - 1}{r} \right) \]

in which:

- \( \rho \) = charge, coul
- \( N_v \) = free ion density, no/m³
- \( e \) = electronic charge, coul
- \( \sigma \) = permittivity of free space, coul²/(N·m³)
- \( b \) = ion mobility, m²/(V·sec)
- \( d \) = thermal velocity, m/sec
- \( r \) = particle radius, m
- \( k \) = Boltzmann's constant, J/K
- \( T \) = temperature, K
- \( K \) = dielectric constant
- \( \rho \) = adjustable parameter

This expression is similar to one developed by Cotton. Although the above expression represents the charging of submicron particles with sufficient accuracy to be useful in calculating precipitator performance, there are indications that the relationships developed by Smith and McDonald describe the charging process more accurately. Therefore, it may be desirable to incorporate their calculation procedure into the model at a later date.

The value of the electric field used for the particle charging calculations is simply the average value between the discharge and collecting electrodes. In order to calculate the velocity of charged particles near the collecting electrode, however, it is necessary to compute the local electric field velocity in this region of space. A review of previous work on this problem indicates that the most promising method of calculation was a numerical integration method.
technique introduced by Leuen and Bäbler. The equations which must be solved are given in two dimensions, written in different forms:

\[
\frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} = -\varepsilon
\]

and

\[
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial S}{\partial x}
\]

where

\( V \) = potential volts

\( \varepsilon \) = mean charge, m/sec

\( \lambda \) = equivalent potential of gas flow from wire to plane

\( Y \) = distance parallel to gas flow from wire to wire

A computer program based on these equations was written and incorporated into the model as a subroutine. In order to check the accuracy of the calculation procedure, the computer program has been used to calculate potential profiles and electric fields based on the geometry and operating conditions given for electrical field measurements reported in the literature.

Figure 1 shows calculations based on the geometry and operating conditions, reported by Pfenley and Metcalf, and their experimental results. Fairly good agreement is found for the potential profiles from the wire to the plate and from a point midway between wires to the plane. Also the field near the plane (the slope of the potential curve) is an excellent agreement. As a result of these and other comparisons between computed and measured results it was concluded that Leuen and Bäbler's technique provides a basis for computing electric fields in the region of interest adjacent to the collecting electrode.

If the particle charge and the electric field adjacent to the collecting electrode have been calculated, the next step in calculating theoretical collection efficiency is the calculation of electrical drift velocity or migration velocity resulting from the electric field and viscous drag forces acting upon a suspended particle. For particle sizes in the size range of interest, the acceleration time is negligible and the migration velocity is given by

\[
\frac{u}{C} = \frac{\varepsilon}{2C}
\]

where

\( u \) = migration velocity of a particle of radius \( r \), m/sec

\( C \) = Cunningham correction factor

\( \varepsilon \) = gas velocity (kg/cm²/sec)

Gas flow velocities in most cases of practical interest are between 0.80 to 1.8 m/sec, while theoretical migration velocities for particles smaller than 1.0 micron externally less than 0.05 m/sec. The path of these smaller particles tends to be disturbed by the turbulent motion of the gas stream in the inter-electrode region. The clearance required for describing particle collection in an electrostatic precipitator under turbulent flow conditions was calculated and gives collection efficiency \( \eta \) as a function of gas volume flow, collection area, and migration velocity:

\[
\eta = 100 \left[ 1 - \exp\left(-\frac{A_0}{\rho_0 U}ight) \right]
\]

where

\( A_0 \) = collection efficiency of a particle of radius \( r \), %

\( U \) = gas volume flow, m/sec

The assumptions on which the derivation of this equation is based:

a) Gas turbulence provides sufficient mixing to establish a uniform particle concentration at any cross section of the precipitator.

b) Turbulent gas streams through the precipitator are uniform across the precipitator and turbulent gas streams through the precipitator are uniform across the precipitator and turbulent gas streams through the precipitator are uniform across the precipitator.

c) The particle migration velocity near the collecting surface is constant for all particles and the average gas flow velocity. This implies that the equation is strictly applicable only to monodisperse particles with a diameter less than about 6 to 80 nm.

February 1975 volume 28, No. 2

Effect of Gas Velocity Distribution

Although it is widely known that a poor velocity distribution gives a lower than anticipated efficiency, it is difficult to apply a
numerical description for gas flow quality. This discussion will describe an approach to the calculation of degradation of performance based on the velocity distribution, the theoretical or ideal efficiency, and the Deutsch equation.

The Deutsch equation can be rearranged to allow calculation of the corrected penetration of a given size particle as a function of the efficiency expected with a uniform velocity and the actual velocity distribution. This can be accomplished as follows:

1. Calculate a constant $k$ from the efficiency predicted under ideal conditions:

$$k = u_0 \ln \frac{1}{1 - q/100} \quad (8)$$

2. Calculate the mean penetration:

$$p = \frac{1}{N_u} \sum u_i (1 - u_i/100) \quad \Rightarrow \quad p = \frac{1}{N_u} \sum u_i u_i \quad (9)$$

where:
- $u_i$ = average velocity, m/sec
- $p$ = corrected penetration fraction of a given size particle
- $N_u$ = number of points or channels with a given velocity
- $u_i$ = point value of velocity
- $u_i$ = point value of efficiency

For any practical velocity distribution and efficiency, the mean penetration obtained by summation over the velocity curves will be higher than the calculated penetration based on an average velocity. If an apparent migration velocity for a given particle size is computed based upon the mean penetration and the Deutsch equation, the result will be a value lower than the value used for calculation of the single point value of penetration. The ratio of the original migration velocity to the reduced migration velocity is a numerical measure of the performance degradation caused by a non-uniform velocity distribution. An expression for this ratio may be obtained by setting the penetration based on the average velocity equal to the corrected penetration obtained from a summation of the point values of penetration, and solving for the required correction factor, which will be a divisor for the migration velocity.

The corrector factor $F$ may be obtained from:

$$\exp \left( -\frac{1}{F \cdot u_i} \right) = \frac{1}{N_u} \sum u_i \exp (-k/u_i) = p \quad (11)$$

Therefore:

$$F = -\frac{1}{k \cdot u_i} \quad (12)$$

Whether the quantity $F$ correlates reasonably well with statistical measure of velocity non-uniformity is yet to be established. A limited number of traverse calculations seem to indicate a correlation between the factor $F$ and the normalized standard deviation of the velocity traverses. Figure 3 shows $F$ as a function of the ideal efficiency for several values of gas velocity standard deviation. These curves were obtained by computer evaluation of equation 12, and the data on which the calculations are based were obtained from Freuler and Lajus.1 The standard deviations have been normalized to represent a fraction of the mean. The overlapping of the curves for standard deviations of 1.0 and 1.00 indicates that the standard deviation above does not completely determine the relationship between $F$ and collection efficiency.

Gas Sneakage and Dust Re-entrainment

Gas sneakage occurs when gas bypasses the electrostatic area of an electrostatic precipitator by flowing through the hopper or through the high voltage insulation space. Streakage can be reduced by frequent baffles which force the gas to enter the main gas passages between the collection plate. If there were no baffles, the percent sneakage would establish the maximum possible penetration because it would be the percent volume having zero collection efficiency. With baffles, the sneakage re-entrains with part of the main flow and then re-by-passes in the next

unbaffled area. The limiting penetration due to sneakage will therefore depend on the amount of sneakage gas per section, the degree of re-making, and the number of sections.

If we make the simplifying assumption that perfect mixing occurs following each baffled section, a simple expression may be derived which relates penetration to the fractional amount of sneakage per section, the ideal efficiency, and the number of stages over which the by-passage is assumed to occur.

$$p_s = \left[ (1 + D) (1 - q/100)^{1/N_u} \right] \quad (13)$$

where:
- $p_s$ = penetration corrected for sneakage
- $D$ = fractional amount of sneakage per section
- $N_u$ = number of baffled sections
- $q$ = collection efficiency of a given particle size obtained with no sneakage.

Journal of the Air Pollution Control Association
Rapping mechanism is concerned with the amount of material that is entrapped by the gas stream after being dislodged from the collection plate by rapping. If we make the simplifying assumption that a fixed fraction of the collected material of a given particle size is entrapped, and that the fraction does not vary with length through the precipitator, an expression can be derived identical in form to that obtained for gas message:

\[ p_s = \left( R - (1 - R) \right) \left( (1 - R) \right) \]  

where:

- \( p_s \) = penetration theoretical for rapping
- \( R \) = fraction of material entrapped per section
- \( \lambda_s \) = number of stages over which rapping is assumed to occur
- \( \eta_s \) = collection efficiency of a given particle size obtained with no rapping.

Figure 3 shows the effect on rapping efficiency for a given particle size of various degrees of rapping; for a four-stage precipitator with the indicated values of re-instrumentation efficiency. Since both the expressions for rapping and message result in a reduction of the expected collection efficiency under ideal conditions, it is possible to define a correction factor for the Drummond equation, which is a divisor for the collection efficiency, analogous to that defined for the effect of nonuniform gas velocity distribution. This expression for the correcting factor is, in terms of rapping:

\[ B = \frac{1}{1 - \lambda_s} \]  

The foregoing expressions for rapping and gas message are overemphasized, but it is believed this analysis will be useful by providing a basis for estimating the order of magnitude of efficiency losses caused by these phenomena. If experimental data on these losses become available, it should be possible to develop more sophisticated models.

Description of Model

The program is structured around three major loops, the determination of which is a direct iteration that converges on the overall mass efficiency. An initial search of overall mass efficiency is refined because the space charge on the particles at any point in the precipitator is a function of the particle charge and the number of particles remaining in the gas. The program contains a calculating procedure which estimates the effect of particle space charge on the average free ion density and the electric field near the collecting electrode. The second major loop includes the calculations which must be performed in each interatomic length, and the nextor allows calculating the calculations dependent on particle size. After the theoretical collection efficiencies for each particle size have been obtained, the program calculates an effective migration velocity for each particle size and, thus:

\[ \eta_s = \frac{1}{1 - R} \]  

where \( \eta_s \) is again the collection efficiency of the particle size under consideration. At this point, a table of ideal or theoretical efficiencies and effective migration velocities is available for the representative particle sizes in the histogram of the particle size distribution. The program then evaluates the correction factors for gas velocity and re-instrumentation, and calculates reduced effective migration velocities and collection efficiencies for each particle size. Overall mass efficiency is obtained by summing over the particle size distribution.

Results

For the purpose of developing typical performance relationships for electrostatic precipitators consisting of a representative particle size distribution and secondary voltage vs current relationship were chosen for input data to the computer model. Operating current density from 3 to 40 na/cm² were selected. Theoretical values of effective migration velocity as a function of particle size are given in Figure 8. The values shown were computed with a specific collection area of 200 ft²/1000 cfm. Particle changing dynamics becomes significant at the lower current densities, and the indicated value for effective migration velocities would decrease somewhat if the program was premised with a lower value of specific collecting area. If the previously discussed equations are utilized in the computer model, the predicted values of overall efficiency are reduced to the range of values obtained from field measurements.

February 1975 Volume 25, No. 2

Figure 8. Theoretical effective migration velocity as a function of current density for particle size.
Figures 9 and 10. These calculations are based on the assumption that the change in mean charge suppression of a given current with size distribution is not a significant factor. This assumption is valid only if the dust loading in the fine particle range is not unusually large.

Log normal particle size distributions with mass median diameters of 25, 10, 5, and 3 μm and a geometric standard deviation of 2.8 were used as input data to the computer model along with a current density of 20 m A/cm², a gas velocity, standard deviation of 0.66, and a reentrainment-entrainment factor of 0.1 over 4 stages. The results from these computer simulations are given in Figure 9. As would be expected, the computed performance is a strong function of the mean median diameter of the distribution.

It is of interest to examine the variation in predicted performance caused by varying the standard deviation of a log normal size distribution with a given mass median diameter. Figure 10 presents results from computer simulations using a log normal particle size distribution with a mean median diameter of 10.0 μm and standard deviations ranging from 1.0 (a mono-disperse distribution) to 5.0. Figure 10 indicates that predicted performance decreases with increasing values of particle size standard deviation. This decrease results from the influence of

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Journal of the Air Pollution Control Association
the increasing proportions of fine particulate which are present with the larger values of standard deviation. Note that the use of a monodisperse distribution with a diameter of 10.0 µm gives results vastly different from those obtained with realistic value of standard deviation.

Conclusions
Calculation of overall collection efficiency of polydisperse particulate in an electrostatic precipitator from theoretical relationships gives results considerably higher than those obtained from performance measurements on full-scale units for coal-fired power boilers. Comparisons to the idealized or theoretical collection efficiency to estimate the effects of non-uniform gas flow, rapping redistribution, and gas by-passing the electrified range of values obtained from field measurements. These calculations suggest that the theoretical model may be used as a basis for quantifying performance under field conditions if sufficient data on the major non-idealities become available.

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References

Dr. Gosch is head of the Chemical Engineering Section, and Mr. Francis is a research engineer in the Engineering and Applied Sciences Department, Southern Research Institute, 2000 Ninth Avenue South, Birmingham, Ala. 35205.

Adhesive Behavior of Dust in Electrostatic Precipitation

Gaylord W. Penney
Carnegie-Mellon University

In 2-stage precipitators particles are driven to the collecting plates by electrostatic forces but then the electrostatic force reverses and tends to pull the particles off so that dust is held on the collecting electrodes only by adhesion. In Cottrell or single-stage precipitators the corona current can provide a significant force tendancy to hold the collected dust to the electrode provided that the resistivity of the dust is 10^8 ohm-cm or more. Adhesion is still essential in the collection of lower resistivity dust and is of vital importance in the transfer of dust from the collecting electrodes to the hopper. As the dust falls from the plates to the hopper it must be held in agglomerations or chunks. There are many peculiarities in the adhesive behavior of electrostatically collected dust. A better understanding of this adhesive behavior is essential if we are to improve the transfer of dust from the collecting electrodes to the hopper.

February 1975 Volume 25, No. 2