1. A novel reactor used in special processing operations is the foam reactor. This reactor is basically a PFR, but the fluid pumped is a foam (liquid-gas mixture). Assuming that the reaction occurs only in the liquid phase of the foam, derive the differential mole balance for the reactor in terms of:

- \( r_A \) = rate of reaction, g-mol A per cm³ of liquid per second
- \( e \) = volume fraction of gas
- \( F_A \) = molar flow rate of A, g-mol/s
- \( V \) = volume of reactor

\[ F_A \rightarrow \text{PFR} \rightarrow F_A \]

Start with a differential mole balance around a fluid element, \( \Delta V \):

\[ \text{In} - \text{out} + \text{Gen} = \frac{d\text{In}}{dt} \]  
\[ F_j(x) - F_j(x + \Delta x) + r_j \Delta V = 0 \]  
(In general)

In this case, only liquid can react, so actual vol of reactant is \((1-e)V\).

\[ \Delta V_j = \frac{F_j(x + \Delta x) - F_j(x)}{r_j} \]

\[ A_{r_j} = \lim_{x \to 0} \frac{F_j(x + \Delta x) - F_j(x)}{\Delta x} \]

\[ \frac{dF_j}{dx} = A_{r_j} \]

\[ dF_j = r_j \Delta V = \frac{dE_j}{dt} \]

\[ dF_j = r_j (1-e) \Delta V \]

\[ \frac{dE_j}{dt} = (1-e) r_j \]
2. Do problem P1.10 (a-g) in Fogler—The LA Basin Problem

P1-10 a

\[ V : 4 \times 10^{13} \text{ ft}^3 \quad \text{Assume CSTR conditions} \]

CO info:
400,000 cars
3000 ft^2 exhaust / car / hr
\( v_0 = \text{ constant} \)

\[ 2.04 \times 10^{10} \text{ lbmol} / \text{ft}^3 \]

\[ 20^\circ \]

\( F_{CO} \)

\[ V = 4 \times 10^{13} \text{ ft}^3 \]
\[ T = 75^\circ \text{F} \]
\[ P = 1 \text{ atm} \]
\[ R = 0.73 \text{ ft}^2 \text{ atm} / \text{ lbmol} / \text{hr} \]

\[ \frac{n}{RT} = \frac{(1 \text{ atm})(4 \times 10^{13} \text{ ft}^3)}{0.73 \text{ ft}^2 \text{ atm} / \text{ lbmol} / \text{hr}} = 1.025 \times 10^{11} \text{ lbmol} \]

b) What is the rate, \( F_{CO, A} \) in \( \text{lbmol CO/hr} \)

\[ \frac{\text{lbmol CO}}{\text{hr}} = \frac{0.02 (3000 \text{ ft}^2)}{\text{hr car}} \times \frac{400,000 \text{ cars}}{\text{hr}} = 2.40 \times 10^7 \text{ ft}^3 \]

\[ v_1 = \frac{(1 \text{ atm})(2.14 \times 10^8 \text{ ft}^3 / \text{hr})}{(0.73 \text{ ft}^2 \text{ atm} / \text{ lbmol} / \text{hr})} = 6.15 \times 10^4 \text{ lbmol} / \text{hr} \]

\[ F_{CO, A} = 6.15 \times 10^4 \text{ lbmol CO/hr} \]
c) What is \( \tau \) (ft²/lb) of a 15 mph wind through 2000 ft.

\[
\tau = \frac{W}{A_{20}} = \frac{5280 \text{ ft}}{20 \text{ mi}} \cdot \frac{79200 \text{ ft/hr}}{1 \text{ hr}}
\]

\[
A_{20} = 20 \text{ m}^2 = 1500 \text{ ft}^2 \cdot \frac{2000 \text{ ft}}{1 \text{ m}} = 2.11 \times 10^5 \text{ ft}^2
\]

\[
\tau = \frac{5280 \times 79200}{1500 \times 2.11 \times 10^5} = 1.67 \times 10^{-12} \text{ ft}^3/\text{sec}^2
\]

d) \( F_{in} = ? \)

\[
C_{in} = 2.04 \times 10^{-10} \text{ kmol/m}^3
\]

\[
\nu = 1.67 \times 10^{-13} \text{ cm}^3/\text{mole}
\]

\[
\nu = 2.04 \times 10^{-10} \text{ kmol/m}^3 \cdot \frac{1.67 \times 10^{-13} \text{ ft}^3}{1 \text{ kmol}} = 3.407 \text{ kmol/hr}
\]

e) Local unsteady mole balance on CO

\[
\begin{align*}
\text{In} - \text{Out} + \text{Gen} &= 0 \quad \text{Take CO from autocatalyst on inlet flow} \\
F_{in} + F_{R,\text{CO}} - V_0 C_{\text{CO}} &= \frac{dN_{\text{CO}}}{dt} \\
N_{\text{CO}} &= C_{\text{CO}} \cdot V \\
F_{out} + F_{R,\text{CO}} - V_0 C_{\text{CO}} &= \frac{d(C_{\text{CO}} \cdot V)}{dt} \\
F_{in} + F_{R,\text{CO}} - V_0 C_{\text{CO}} &= \frac{dC_{\text{CO}}}{dt}
\end{align*}
\]
\( F_{in} + F_{in} - V_oC = V \frac{dc}{dt} \)

\( \frac{dc}{dt} + \frac{V_o}{V} c = F_{in} \frac{F_{in}}{V} \quad \text{let } R = \frac{V_o}{V} \quad \text{and } K = \frac{F_{in} F_{in}}{V} \)

\( \frac{dc}{dt} + RC = K \quad \text{Integrating factor } = e^{\frac{Rt}{RC}} = e^{\frac{t}{R}} \)

\( \frac{d}{dt} (e^{\frac{Rt}{RC}} c) = Ke^{\frac{Rt}{RC}} \)

\( e^{\frac{Rt}{RC}} c = K \int e^{\frac{Rt}{RC}} dt \)

\( e^{\frac{Rt}{RC}} c = K \left[ e^{\frac{R}{RC} t} + ce^{-\frac{R}{RC} t} \right] \)

\( c = \frac{k}{R} + kc e^{-\frac{R}{RC} t} \)

\( c = K \left[ \frac{1}{R} + kc e^{-\frac{R}{RC} t} \right] \)

\( c_0 = \frac{K}{R} + kc \)

\( c = \frac{c_0}{K} e^{-\frac{R}{RC} t} \)

\( \frac{c}{K} = \frac{c_0}{R} \quad \text{plug in } c_0 \quad F_{in} + F_{in} - V_oC \)

\( t = \frac{V}{V_o} \ln \left( \frac{F_{in} F_{in} - V_oC}{F_{in} F_{in} - V_oC} \right) \)
\[ C_0 = 2.04 \times 10^{-8} \text{ lb mol/ft}^3 \]

Find how long for \( C = 2 \text{ ppm} \) or \( 5.0 \times 10^{-9} \text{ lb mol/ft}^3 \).

\[ t = \left( \frac{4 \times 10^3 \text{ ft}^3}{1.67 \times 10^5 \text{ ft}^3/\text{hr}} \right) \ln \left[ \frac{6.15 \times 10^4 \text{ lb mol/hr} + 3.46 \times 10^7 \text{ lb mol/hr}}{1.67 \times 10^5 \text{ lb mol/hr}} \right] \\
= 2.4 \ln \left( \frac{6.15 \times 10^4 - 2 \times 10^6}{6.15 \times 10^7} \right) \\
= 2.4 \ln (14.8) \\
= 6.5 \text{ hr} \]
3. [Former Exam Problem] You've rented a cheap hotel room that reeks of cigarettes and booze and you want to air it out by running the room's ventilation system. The room is 8 meters long, 5 meters wide, 2.5 meter high, and can be considered well mixed. You determine that the concentration of stink in the air initially is $C_{\text{stink}} = 5 \times 10^{-3}$ mole/m$^3$. It turns out that the carpet on the floor is the source of the smell; it puts out $2.5 \times 10^{-7}$ moles of stink per square meter per second. (a) The high setting on the fan provides 0.1 m$^3$/s of ventilation air flow. What is the steady state stink concentration in the room under these conditions? (b) If you started blowing air at 0.1 m$^3$/s as soon as you entered the room, how much stink would be left in the air after one hour of running the vent?

\[
\text{Volume} = V = 100 \text{ m}^3
\]

\[
\text{Flow rate} = G = 2.5 \times 10^{-7} \text{ mole/m}^2 \text{s}
\]

\[
C_{\text{stink}} = 5 \times 10^{-3} \text{ mol/m}^3
\]

\[\begin{align*}
\text{a) Steady state mole balance on stink} \\
\text{In - Out + Gen = \text{rate}} \\
0 - F_{\text{stink}} + 2.5 \times 10^{-7} \text{ mol/m}^2 \text{s} \frac{100 \text{ m}^3}{\text{s}} &= 0 \\
\Gamma_{\text{stink}} &= 1.0 \times 10^{-5} \text{ mol stink/m}^3 \\
C_{\text{stink}} &= \frac{F_{\text{stink}}}{V} = 1.0 \times 10^{-4} \text{ mol stink/m}^3
\end{align*}\]

\[\begin{align*}
\text{b) Unsteady state mole balance} \\
\text{In - Out + Gen = \text{rate}} \\
0 - V_dC + G &= \frac{V_vC}{dt} \\
\frac{V_vC}{dt} + \frac{V_vC}{V} &= \frac{G}{V} \\
\frac{dC}{dt} + \frac{V_v}{V}C &= \frac{G}{V} \\
\text{Integrating Factor} &= e^{\frac{V_v}{V}t} \\
\frac{d}{dt} (e^{\frac{V_v}{V}t} C) &= e^{\frac{V_v}{V}t} \frac{G}{V} \\
e^{\frac{V_v}{V}t} C &= \int e^{\frac{V_v}{V}t} \frac{G}{V} dt
\end{align*}\]
\[ C(t) = \frac{C_0}{\sqrt{V_0}} e^{-\frac{V}{V_0} t} + C_i \]

\[ C = \frac{C_0}{\sqrt{V_0}} + \frac{C_i}{\sqrt{V}} \]

\[ C(t) = \frac{C_0}{\sqrt{V_0}} \left( 1 - e^{-\frac{V}{V_0} t} \right) \]

\[ \frac{V}{C} \left( C_0 - \frac{C}{\sqrt{V_0}} \right) = C_i \]

\[ C = \frac{C_0}{\sqrt{V_0}} \left[ C_0 - \frac{C_0}{\sqrt{V_0}} \right] e^{-\frac{V}{V_0} t} \]

\[ \frac{C}{V_0} = 1.0 \times 10^{-5} \text{ mol \, m}^{-3}, \quad \frac{1}{0.1 \text{ m}^3/\text{s}} = 1.0 \times 10^4 \text{ mol \, m}^{-3} \text{ s}^{-1} \]

\[ C = 1.0 \times 10^{-4} \text{ mol \, m}^{-3} + \left[ C \times 10^{-5} \text{ mol \, m}^{-3} - 1.0 \times 10^{-4} \text{ mol \, m}^{-3} \right] e^{-\frac{V}{V_0} t} \]

\[ C = 1.0 \times 10^{-4} \text{ mol \, m}^{-3} + \left[ 0.0049 \text{ mol \, m}^{-3} \right] e^{-0.0016 t} \]

\[ t = 3.6005 \text{ s} \text{ after 1 hr} \]

\[ C = 1.0 \times 10^{-4} \text{ mol \, m}^{-3} + 1.34 \times 10^{-4} \text{ mol \, m}^{-3} \]

\[ C = 2.34 \times 10^{-4} \text{ mol \, m}^{-3} \text{ after 1 hr} \]

\[ N = CV = 0.0234 \text{ mols \, m}^{-3} \text{ left in room after 1 hr} \]
4. Use MATLAB to find the root of the following equations (you must turn in a print out of all MATLAB code to receive credit).

(a) $5x^5 + 5x^4 - 5x + 5 = 0$

(b) $12x^7 - 12x^4 - 12x^2 + 12 = 0$

```matlab
>> c = [5 5 0 0 -5 5];
>> roots(c)
ans =
    -1.4971
   -0.3947 + 1.0247i
   -0.3947 - 1.0247i
    0.6433 + 0.3743i
    0.6433 - 0.3743i
```

```matlab
>> d = [12 0 0 -12 0 -12 0 12]

d =

12 0 0 -12 0 -12 0 12

>> roots(d)
ans =
   -0.7678 + 0.8937i
   -0.7678 - 0.8937i
    0.1846 + 1.0106i
    0.1846 - 1.0106i
    1.0000
    0.9135
   -0.7472
```
5. Use MATLAB to solve the following algebraic equations (you must turn in a print out of all MATLAB code to receive credit).

\[ 7x + 3y + 5z = 0 \]
\[ 2x - 3y - 5z = 0 \]
\[ 10x + 3y - z = 0 \]

\[
\begin{bmatrix}
7 & 3 & 5 \\
2 & 3 & -2 \\
10 & 2 & -1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]

\[ \text{rhs} = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix} \]
\[ \text{rhs} = \text{rhs} \]
\[ A = x \backslash \text{rhs} \]
\[ A = \begin{bmatrix}
10 & 0 & 0 \\
0 & 7 & 0 \\
0 & 0 & 2
\end{bmatrix} \]
\[ A = \text{inv}(A) \cdot \text{rhs} = x \backslash \text{rhs} \]

\[ \text{MATLAB Code:} \]

\[
X = [7 3 5; 2 -3 -5; 10 3 -1]
\]
\[ \text{rhs} = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix} \]
\[ \text{Rhs} = \text{Rhs} \]
\[ A = X \backslash \text{Rhs} \]
6. Use MATLAB to solve the following ordinary differential equation (ODE), and compare with an analytical solution (you must turn in a print out of your MATLAB code to receive credit).

\[
\frac{dC}{dt} = -kC,
\]

where \( k = 10 \) s\(^{-1} \), and the initial value for \( C \) is \( C_0 = 1 \) g-mol per liter. How long does it take to exceed 90% conversion? What kind of a reactor does this equation represent?

\[
\frac{dC}{dt} = k \frac{C_0}{C} - 1 \ln \left( \frac{C}{C_0} \right)
\]

\[
t = -\frac{1}{k} \ln \left( \frac{C}{C_0} \right)
\]

\[
t = -\frac{1}{k} \ln \left( \frac{0.1}{C_0} \right) = 0.23 \text{ s}
\]

This is a PFR reactor design equation.

clear
%this program finds the time needed for a reaction to reach 90% conv.
%this program calls function 'ode6'

global k
C0 = 1; k = 10;

C0 = C0;
tfin = 0.5;
%integrate from 0 to t
[t,C] = ode45('ode6', 0, tfin, C0);

plot(t,C)

function dc_dt = ode6(t,C)
global k
dc_dt = -k*C;