One-minute responses

• Q: How does an organism know to produce fewer homozygotes when they are disfavored in an overdominant situation?

• A: It doesn’t know. Offspring are at H-W at conception, but the homozygotes don’t survive as well, so by breeding time there are fewer.

• Q: How do you know which term goes where in $t/(s + t)$?

• A: If you are calculating $pA$, the one on top is the badness of the other allele $a$. 
Linkage of Genes on Chromosomes

1. Finishing up sex linkage
2. Linkage and natural selection
3. Effects of recombination
4. Pros and cons of recombination
5. Linkage disequilibrium
Suppose that among X chromosomes, $p(X^H) = 0.8$ and $p(X^h) = 0.2$ in both sexes.

<table>
<thead>
<tr>
<th>Genotype</th>
<th>$X^H X^H$</th>
<th>$X^H X^h$</th>
<th>$X^h X^h$</th>
<th>$X^H Y$</th>
<th>$X^h Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fitness</td>
<td>1.0</td>
<td>1.0</td>
<td>0.1</td>
<td>1.0</td>
<td>0.1</td>
</tr>
<tr>
<td>HW</td>
<td>0.32</td>
<td>0.16</td>
<td>0.02</td>
<td>0.40</td>
<td>0.10</td>
</tr>
</tbody>
</table>

We can see immediately that a rare recessive sex-linked disease shows up mostly in males.
**Sex linked traits**

<table>
<thead>
<tr>
<th>Genotype</th>
<th>$X^H X^H$</th>
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<th>$X^h X^h$</th>
<th>$X^H Y$</th>
<th>$X^h Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fitness</td>
<td>1.0</td>
<td>1.0</td>
<td>0.0</td>
<td>1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>HW</td>
<td>0.32</td>
<td>0.16</td>
<td>0.02</td>
<td>0.40</td>
<td>0.10</td>
</tr>
<tr>
<td>Post-Selection</td>
<td>0.36</td>
<td>0.19</td>
<td>0.0</td>
<td>0.45</td>
<td>0.0</td>
</tr>
</tbody>
</table>

The new allele frequencies are:

Females: $p(X^H) = 0.91$, $p(X^h) = 0.09$

Males: $p(X^H) = 1.0$, $p(X^h) = 0.0$
Sex linked traits are weird

- Even if selection stops, system won’t go straight to HW:
  - Mating is non-random with respect to sex
  - Males and females have different allele frequencies
**X linked traits**

- X-linked recessive decreases faster than an autosomal recessive
- Exposed to selection when in males
- Sex-linked traits don't go to Hardy-Weinberg in one generation
  - Without selection, they go to Hardy-Weinberg slowly over many generations
  - With selection, they may never get there
Sex linked traits

- A point to bear in mind:
  - Most sex-specific traits are not sex-linked (on X or Y)
  - Most sex-linked traits (on X) are unrelated to sex
  - Examples: hemophilia, color vision
  - The Y chromosome contains a few “switch” genes which control sex in humans
  - Almost all of the genes controlled by these switches are autosomal

- Why?
The only Y-linked non-sex gene I know of
Sex linked traits

• Why aren’t sex-related traits sex-linked?

• Both males and females have X

• Why aren’t male traits on the Y?
  – If sex-related traits evolved from other traits, they would start off on the autosomes
  – The Y is haploid and mostly non-recombining, which can cause its genes to deteriorate
  – Having one master switch rather than many independent sex-related trait genes may be less fragile
Selection at multiple loci, no linkage

- IF loci are unlinked (e.g. different chromosomes, or far apart)
- AND phenotypes don’t interact
- THEN each locus experiences selection independently
Example with unlinked loci

Two unlinked loci A and B with non-interacting phenotypes:

<table>
<thead>
<tr>
<th>Genotype</th>
<th>AA</th>
<th>Aa</th>
<th>aa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fitness</td>
<td>1.0</td>
<td>1.0</td>
<td>0.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Genotype</th>
<th>BB</th>
<th>Bb</th>
<th>bb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fitness</td>
<td>0.8</td>
<td>1.0</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Assume that an organism’s fitness is the product of its fitness at each locus

- What will happen at locus A?
- What will happen at locus B?
Linkage in Organisms Without Recombination

In an organism without recombination, if two mutations occur on different copies of a chromosome, they cannot be brought together.

--------A--------b--------

--------a--------B--------

• If they occurred on the same copy, they can’t be separated either

• The only way to break up combinations is repeated mutation
Linkage in Organisms Without Recombination

\[ -------A-------b------- \]
\[ -------a-------B------- \]

If both \( A \) and \( B \) are advantageous alleles, this linkage blocks the population from reaching maximum fitness.

- May remain polymorphic (both alleles present)
- One may eliminate the other
- In either case, maximum fitness cannot be reached
**Practical examples**

<table>
<thead>
<tr>
<th>Genotype</th>
<th>AA</th>
<th>Aa</th>
<th>aa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fitness</td>
<td>1.0</td>
<td>1.0</td>
<td>0.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Genotype</th>
<th>BB</th>
<th>Bb</th>
<th>bb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fitness</td>
<td>1.0</td>
<td>1.0</td>
<td>0.7</td>
</tr>
</tbody>
</table>

If the population contains only Ab and aB gametes, and there is no recombination, what will selection do?

Assume allele frequencies of $p(A) = 0.6$, $p(B) = 0.4$, and that an organism’s fitness is the product of its fitness at the two loci.

What will happen in the short term? In the long term?
Practical examples

The population really behaves like this:

<table>
<thead>
<tr>
<th>Genotype</th>
<th>Ab/Ab</th>
<th>Ab/aB</th>
<th>aB/aB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fitness</td>
<td>0.7</td>
<td>1.0</td>
<td>0.7</td>
</tr>
<tr>
<td>Frequency</td>
<td>0.36</td>
<td>0.48</td>
<td>0.16</td>
</tr>
</tbody>
</table>

This will move to an equilibrium $p(A)=p(B)=0.5$ and stay there. Even though it contains two loci with clear-cut bad recessives, it behaves like one overdominant locus.
Practical examples

Now add recombination of 10% between the two loci.

\[ p(\text{Ab}) = 0.6, \quad p(\text{aB}) = 0.4 \]

Gametes after 1 generation:

90% are non-recombinant, producing 0.54 Ab and 0.36 aB.

10% are recombinant, and will therefore be assorted at random: 0.024 AB, 0.036 Ab, 0.016 aB, 0.024 ab.

Totals: 0.024 AB, 0.576 Ab, 0.376 aB, 0.024 ab

A Punnett Square is the easiest way to proceed from here.
## Practical examples

<table>
<thead>
<tr>
<th></th>
<th>AB 0.024</th>
<th>Ab 0.576</th>
<th>aB 0.376</th>
<th>ab 0.024</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB 0.024</td>
<td>0.0006</td>
<td>0.0138</td>
<td>0.0090</td>
<td>0.0006</td>
</tr>
<tr>
<td>Ab 0.576</td>
<td>0.0138</td>
<td>0.3318</td>
<td>0.2166</td>
<td>0.0138</td>
</tr>
<tr>
<td>aB 0.376</td>
<td>0.0090</td>
<td>0.2166</td>
<td>0.1414</td>
<td>0.0090</td>
</tr>
<tr>
<td>ab 0.024</td>
<td>0.0006</td>
<td>0.0138</td>
<td>0.0090</td>
<td>0.0006</td>
</tr>
</tbody>
</table>
**Practical examples**

<table>
<thead>
<tr>
<th>Genotype</th>
<th>AABB</th>
<th>AABb</th>
<th>AAbb</th>
<th>AaBB</th>
<th>AaBb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freq</td>
<td>0.0006</td>
<td>0.0276</td>
<td>0.3318</td>
<td>0.0180</td>
<td>0.4344</td>
</tr>
<tr>
<td>Fitness</td>
<td>1.0</td>
<td>1.0</td>
<td>0.7</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Genotype</th>
<th>Aabb</th>
<th>aaBB</th>
<th>aaBb</th>
<th>aabb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freq</td>
<td>0.0276</td>
<td>0.1414</td>
<td>0.0180</td>
<td>0.0006</td>
</tr>
<tr>
<td>Fitness</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
<td>0.49</td>
</tr>
</tbody>
</table>

Now selection has something to work on, and \( p(a) \) and \( p(b) \) will go down. Recombination has converted one “overdominant locus” to two dominant/recessive loci.
Evolutionary consequences of recombination

- **Good:**
  - Avoid “Muller’s Ratchet”
  - Avoid hitchhiking
  - Rapidly combine favorable mutations
  - Increase offspring diversity
  - Repair damaged DNA

- **Bad:**
  - Break up favorable combinations
  - Illegitimate recombination can damage chromosomes
  - Costs energy
Apparently recombination is not essential

Bdelloid rotifer. The entire class is believed to have gone without sex for at least 40 million years (maybe as long as 100 million years). They are common and widespread with at least 360 different species.
Evidence for Asexuality of Bdelloids

- Chromosomes do not match up in pairs
- No males; no meiosis; eggs produced by mitosis
- Relationships between individuals form a simple tree
- Very few active transposable elements

What’s risky about this lifestyle?
Muller’s Ratchet

- Without recombination:
  - All mutation-free copies of a chromosome might be lost
  - Once lost, they are hard to get back
- (A "ratchet" is a gear that turns only one way)
- Muller thus predicts that an asexual genome will deteriorate over time
Muller’s Ratchet

What can counteract the ratchet?

- Back mutation
- New favorable mutations
- Strong selection
- Recombination—two bad chromosomes can recombine into a better one
Bdelloids and their lack of transposons

• Transposons can be costly:
  – Slower DNA replication
  – Production of useless transposase
  – Genomic damage due to transposition

• Transposons tend to increase

• A sexual organism can recombine two transposon-ridden chromosomes and perhaps produce a transposon-free one

• So transposons might be too dangerous for bdelloids to tolerate
How do bdelloids survive without sex?

- Bdelloid lifestyle is prone to desiccation
  - They are highly mutation-resistant (to survive this)
  - They pick up DNA from the environment when dried out
- This may pre-adapt them to living without sex
  - Low mutation rate slows the ratchet
  - Pick-up of foreign DNA allows occasional recombination
Hitchhiking

- A good mutation arises and will sweep the population
- If there were any mildly bad mutations on that chromosome, they will sweep too ("hitchhiking")
- Recombination allows hitchhikers to be removed
Diverse offspring

- Recombination creates novel gene combinations

- If offspring diversity is favorable, recombination helps

- Many organisms use sexual reproduction to produce:
  - Dispersal forms
  - Over-wintering forms
Disequilibrium

- Linkage disequilibrium (LD) is association between alleles at different loci

- Examples:
  - Only AB and ab are present
  - All four combos are present but some are disproportionately common

- Linkage equilibrium is lack of association:
  - You can predict $p_{AB}$ as $(p_A)(p_B)$

- With recombination, population moves towards equilibrium
Disequilibrium

An example:

If we know the allele frequencies:

\[
\begin{align*}
p_A &= 0.7 \\
p_a &= 0.3 \\
p_B &= 0.2 \\
p_b &= 0.8
\end{align*}
\]

then if we have linkage equilibrium, we can immediately predict the proportion of gametes:

\[
\begin{align*}
p_{AB} &= 0.7 \times 0.2 = 0.14 \\
p_{Ab} &= 0.7 \times 0.8 = 0.56 \\
p_{aB} &= 0.3 \times 0.2 = 0.06 \\
p_{ab} &= 0.3 \times 0.8 = 0.24
\end{align*}
\]
The disequilibrium coefficient $D$ is defined as:

$$D = (p_{AB} \times p_{ab}) - (p_{Ab} \times p_{aB})$$

$p_{A} = 0.7$
$p_{a} = 0.3$
$p_{B} = 0.2$
$p_{b} = 0.8$

It can be interpreted as the excess of $AB$ and $ab$ over the other two. If we have $D=0.01$:

$$p_{AB} = 0.7 \times 0.2 + 0.01 = 0.15$$
$$p_{Ab} = 0.7 \times 0.8 - 0.01 = 0.55$$
$$p_{aB} = 0.3 \times 0.2 - 0.01 = 0.05$$
$$p_{ab} = 0.3 \times 0.8 + 0.01 = 0.25$$
Practice problem

pA = 0.7
pa = 0.3
pB = 0.2
pb = 0.8
(just like before)

If D=-0.01, what are the genotype frequencies?
Practice problem

\[ pA = 0.7 \]
\[ pa = 0.3 \]
\[ pB = 0.2 \]
\[ pb = 0.8 \]
If \( D = -0.01 \), what are the genotype frequencies?

\[ pAB = 0.7 \times 0.2 - 0.01 = 0.13 \]
\[ pAb = 0.7 \times 0.8 + 0.01 = 0.57 \]
\[ paB = 0.3 \times 0.2 + 0.01 = 0.07 \]
\[ pab = 0.3 \times 0.8 - 0.01 = 0.23 \]
Disequilibrium

The disequilibrium coefficient $D$ can range between $-0.25$ and $+0.25$. Zero is linkage equilibrium.

Only two loci with the same allele frequencies can reach $-0.25$ or $+0.25$.

Many papers use a normalized coefficient $D'$ which divides by the maximum disequilibrium possible for those allele frequencies. This is easier to compare among loci.
Disequilibrium

Where does disequilibrium come from?

- Newly arisen mutation
- Mixing of population with different alleles
- Selection for specific combinations
- Drift (usually won’t produce very much)
Wild-type chromosomes

New mutation at the disease locus

There now is association between the A and D alleles

Many generations

Few generations

Short linkage disequilibrium

Long linkage disequilibrium
Disequilibrium

• Joe Felsenstein showed in 1988 that a key effect of sexual reproduction and recombination is to reduce linkage disequilibrium

• In asexuals, only back mutation and repeated occurrence of the same mutation can break down disequilibrium

• In sexuals, chromosome segregation and recombination also break down disequilibrium
A human example

Remember the DuffyO allele from HW1?

- Probably (almost) all current DuffyO descend from 1 copy of chr. 1 3000-5000 years ago

- Without recombination every African would have THAT WHOLE CHROMOSOME

- If anything bad was on it, every African would have that too (or it would have stopped DuffyO from fixing)

- With recombination, Africans share only a few kb on either side of Duffy
Effects of Recombination

- Recombination might break up cooperative groups of alleles
- Bacterial genomes often have related genes close together:
  - To reduce the impact of recombination?
  - To make coordinated regulation easier?
- Eukaryotes have less clustering, but sometimes related genes are still found together.
Figure 1. Gene map of the human leukocyte antigen (HLA) region. The HLA region spans $4 \times 10^6$ nucleotides on chromosome 6p21.1 to p21.3, with class II, class III and class I genes located from the centromeric (Cen) to the telomeric (Tel) end. HLA class I molecules restrict CD8+ cytotoxic T lymphocyte function and mediate immune responses against ‘endogenous’ antigens and virally infected targets, whereas HLA class II molecules are involved in the presentation of ‘exogenous’ antigens to T helper cells. The HLA class III region contains many genes encoding proteins that are unrelated to cell-mediated immunity but that nevertheless modulate or regulate immune responses in some way, including tumour necrosis factor (TNF), heat shock proteins (Hsps) and complement proteins (C2, C4) (fig001nmn).
Effects of Recombination

- MHC genes tightly linked in many vertebrates
- MHC linkage disequilibrium high in mice and humans, probably elsewhere too
- Is the linkage important or a historical accident?
- Is the LD important or a historical accident?
- (I don’t know the answer)
LD graphs

• Graphs like this plot $D'$ (or the similar statistic $r^2$) between sites

• In humans, conspicuous blocks of LD are usually seen

• Different ideas about their origin:
  – Recombination hotspots (at edges of blocks)
  – Population admixture followed by a few recombinations
  – Random chance

• I wish we could apply more statistics to this graph!
One-minute responses

- Tear off a half-sheet of paper

- Write one line about the lecture:
  - Was anything unclear?
  - Did anything work particularly well?
  - What could be better?

- Leave in front on your way out