**Problem 1: ER Example**

Consider the emergency room example we studied in class, with a minor modification:

- Emergency cases arrive according to a Poisson process (exponential interarrival times) and that the time spent with the ER doctor is exponentially distributed
- Average arrival rate \( \lambda = 2 \) per hour
- Average service time = \( \mu = 4 \) per hour (this was 3 per hour in class)
- Utilization ratio = \( \rho = \frac{\lambda}{\mu} = 0.5 \) (and this was 2/3)

Remember that we use insights from the birth-and-death modeling of an M/M/1 system, and can obtain formulae for many indicators, such as the expected number of customers in the system, etc.

Download the template on the class website (sheet ‘Excel M/M/S Template’), and using the M/M/1 formulae, experiment with it to find \( P_n \) (for \( n=0,1,2 \ldots \)), \( L, L_q, W, W_q \), \( P \{ \omega > t \} \) and \( P \{ \omega_q > t \} \). Shaded cells are where you would enter your formula.
As a reminder, here are the M/M/1 formulae:

\[
\begin{align*}
    P_n &= (1-\rho)\rho^n \\
    L &= \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda} \\
    W &= \frac{1}{\mu-\lambda} \\
    W_q &= \frac{\lambda}{\mu(\mu-\lambda)} \\
    L_q &= \frac{\lambda^2}{\mu(\mu-\lambda)} \\
    P\{\omega>t\} &= e^{-\mu(1-\rho)t} \\
    P\{\omega_q>t\} &= \rho e^{-\mu(1-\rho)t}
\end{align*}
\]

Answer the following questions using your Excel sheet. For your reference, the filled-in sheet for the values is provided.
1. What is the probability that the doctor is idle?

2. What is the utilization of the doctor (proportion of time the doctor is busy)?

3. What is the probability that there are exactly 10 patients in the ER?

4. What is the expected number of patients in the ER?

5. What is the expected number of patients waiting for the doctor?

6. What is the expected time in the ER?

7. What is the expected waiting time for the doctor?

8. What is the probability that there are at least two patients waiting for the doctor?

9. What is the probability that a patient waits for the doctor more than 30 minutes?

How would your answers change if the average arrival rate was increased to 3 per hour?

1. 2. 3. 4. 5.

6. 7. 8. 9.

What would happen if the arrival rate was 4 per hour?
Problem 2: ER M/M/s

Now consider the M/M/s case of the ER example with two ER doctors (s=2) and three ER doctors (s=3). The arrival rate and service rate are the same as provided in Problem 1:

- Average arrival rate = \( \lambda = 2 \) per hour
- Average service time = \( \mu = 4 \) per hour

Use the template to find the following performance measurements for s=2 and s=3. Compare your results with s=1.

<table>
<thead>
<tr>
<th>Performance Measurements</th>
<th>s=1</th>
<th>s=2</th>
<th>s=3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( L )</td>
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<td>( L_q )</td>
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<td>( W )</td>
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<tr>
<td>( W_q )</td>
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<tr>
<td>P(at least two patients waiting in queue)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>P(a patient waits more than 30 minutes)</td>
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