Case 17.1

a) Status quo at the presses:

Queuing model: M/ M/ s

\[ s = 10, \lambda = 7 \text{ / hour}, \mu = 1 \text{ / hour} \]

In-process inventory: \( L = 7.52 \)

Status quo at the inspection station:

Queuing model: M/ D/ 1

\[ s = 1, \lambda = 7 \text{ / hour}, \mu = 8 \text{ / hour} \]

In-process inventory: \( L = 3.94 \)

Machine cost =($7/hour) (10) = $70 / hour

Inspector cost = $17/hour

\[ E(SC) = 70 + 17 = \$87 / \text{hour} \]

\[ E(WC) = \text{Inventory cost} = (7.52 + 3.94)(\$8/\text{hour}) = \$91.68 / \text{hour} \]

\[ E(TC) = E(SC) + E(WC) = \$178.68/ \text{hour} \]
Case 17.1 (cont’d)

b) Proposal 1: reduce power at presses

Queuing model for the presses: M/M/s

\[ s = 10, \lambda = 7 \text{ / hour}, \mu = 0.833 \text{ / hour} \]

In-process inventory at the presses: \( L = 11.05 \)

In-process inventory at the inspection station: \( L = 3.94 \)

Machine cost = \((10)(\$6.50) = \$65 \text{ / hour}\)
Inspector cost = \$17 \text{ / hour}\)
\( E(SC) = 65 + 17 = \$82 \text{ / hour} \)
\( E(WC) = \text{Inventory cost} = (11.05 + 3.94)(\$8\text{/hour}) = \$119.92 \text{ / hour} \)
\( E(TC) = E(SC) + E(WC) = \$201.92 \text{ / hour} \)

The total cost is higher than for the status quo (Slowing down the machines won’t change in-process inventory for the inspection station.)
c) Proposal 2: Substitute a younger inspector, with original speed

In-process inventory at the presses: $L = 7.52$

Queuing model for the inspection station: $M / E_2 / 1$

$s = 1, k = 2, \lambda = 7 / \text{hour}, \mu = 8.333 / \text{hour}$

In-process inventory at the inspection station: $L = 4.15$

Machine cost = (10)($7/hour) = $70 / hour
Inspector cost = $19 / hour
$E(\text{SC}) = 70 + 19 = $89 / hour$
$E(\text{WC}) = \text{Inventory cost} = (7.52 + 4.15)($8/hour) = $93.36 / hour$
$E(\text{TC}) = E(\text{SC}) + E(\text{WC}) = $182.36 / hour$

The total cost is higher than for the status quo due to the increase in the service rate variability and the resulting increase in the in-process inventory.
They should consider *increasing* power to the presses (increasing their cost to $7.50 per hour while reducing their average time to form a wing section to 0.8 hours).

Queuing model for the presses: M / M / s

\[ s = 10, \lambda = 7 / \text{hour}, \mu = 1.25 / \text{hour} \]

In-process inventory at the presses: \( L = 5.69 \)

With original inspector: In-process inventory at the inspection station: \( L = 3.94 \)

Machine cost = \((10)(\$7.50/\text{hour}) = \$75 / \text{hour}\)
Inspector cost = \$17 / \text{hour}\)
\[ E(SC) = 75 + 17 = \$92 / \text{hour} \]
\[ E(WC) = \text{Inventory cost} = (5.69 + 3.94)(\$8/\text{hour}) = \$77.04 / \text{hour} \]
\[ E(TC) = E(SC) + E(WC) = \$169.04 / \text{hour} \]
Case 2. The PCB production machine repair shop

a) Queuing model: $M / M / s / / / N$

$s = 3, N = 10, \lambda = \frac{1}{5}, \mu = \frac{1}{3}$

\[
P_0 = \frac{1}{\left[ \sum_{n=0}^{2} \frac{10!}{(10-n)!n!} \left( \frac{\lambda}{\mu} \right)^n + \sum_{n=3}^{10} \frac{10!}{(10-3)!3!3^{n-3}} \left( \frac{\lambda}{\mu} \right)^n \right]}
\]

\[
P_n = \frac{10!}{(10-n)!n!} \left( \frac{\lambda}{\mu} \right)^n P_0 \quad \text{for } n = 1, 2, 3
\]

\[
= \frac{10!}{(10-n)!3!3^{n-3}} \left( \frac{\lambda}{\mu} \right)^n P_0 \quad \text{for } n = 4, 5, 6, \ldots, 10
\]
a) (cont'd)

\[ P_0 = 0.0042, P_1 = 0.0252, P_2 = 0.068, P_3 = 0.109, P_4 = 0.152, \]
\[ P_5 = 0.183, P_6 = 0.183, P_7 = 0.146, P_8 = 0.088, P_9 = 0.035, \]
\[ P_{10} = 0.007 \]

\[ L_q = \sum_{n=3}^{10} (n - 3)P_n = 2.349 \]

\[ L = \sum_{n=0}^{2}nP_n + L_q + 3 \left( 1 - \sum_{n=0}^{2} P_n \right) = 5.218 \]
Case 2 (cont’d)

\[ E(\text{# of PCB machines available}) = 10 - L = 4.782 \]

\[ E[\text{revenue}] = E[\text{# of machines available}] \times 50 \times 180 \times 30 = $1,291,075 \text{ per month} \]

\[ E[\text{cost}] = E[\text{# of machines available}] \times 30 \times 50 \times 120 + 90,000 \times 3 = $1,130,717 \text{ per month} \]

\[ E[\text{profit}] = E[\text{revenue}] - E[\text{cost}] = $160,358 \text{ per month} \]
b) Consider leasing another repair station

Queuing model: M / M / s / / / N

\[ s = 4, \quad N = 10, \quad \lambda = \frac{1}{5}, \quad \mu = \frac{1}{3} \]

\[ L = 4.311 \]

\[ E[\text{revenue}] = E[\# \text{ of machines available}] \times 50 \times 180 \times 30 = 5.689 \times 50 \times 180 \times 30 = 1,536,030 \]

\[ E[\text{cost}] = E[\# \text{ of machines available}] \times 30 \times 50 \times 120 + 90,000 \times 4 = 1,384,020 \]

\[ E[\text{profit}] = E[\text{revenue}] - E[\text{cost}] = 152,010 \leq 160,358 \]

No, they should not lease an additional repair station
c) Consider a new type of repair station

Queuing model: M / M / s / / / N

\[ s = 1, 2, 3, 4, 5 \quad N = 10, \lambda = \frac{1}{10}, \mu = \frac{1}{5} \]

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<th>L</th>
<th>E(number of machines available)</th>
<th>E[revenue]</th>
<th>E[cost]</th>
<th>E[profit]</th>
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