Strength Theories

The majority of material strength data is based on uniaxial tensile test results. Usually, all that you have to work with is the yield strength $S_y$ and/or the ultimate tensile strength $S_u$.

This is fine if you only have the one normal stress component present: this is true for simple tension or compression members and for parts loaded only in bending.

$$\sigma_1 = \sigma_x$$

In this case, failure (defined as the onset of plastic deformation) occurs when

$$\sigma_x = \sigma_1 = \frac{S_y}{n}$$

‘$n$’ is the factor of safety.

In many loading cases, we have more than just one normal stress component.

E.g. in torsion, we have a single shear stress component:

$$\tau_{xy}$$

Or, combined bending and torsion in a shaft:

These cases can all be reduced to a simple biaxial case by finding the principal stresses, $\sigma_1$ and $\sigma_2$

Now when does failure occur? For ductile materials there are two commonly used strength theories - the Maximum Shear Stress (MSS) or Tresca theory and the von Mises or Distortion Energy theory.
Strength Theories

1. Maximum Shear Stress:

This states that failure occurs when the maximum shear stress in the component being designed equals the maximum shear stress in a uniaxial tensile test at the yield stress:

This gives \( \tau_{\text{max}} = S_y/2n \) or

\[
| \sigma_1 - \sigma_2 | = S_y/n \\
| \sigma_2 - \sigma_3 | = S_y/n \\
| \sigma_3 - \sigma_1 | = S_y/n
\]

whichever of the last three leads to the safest result. The latter usually involves \( \sigma_3 \) being zero, i.e. plane stress, and both \( \sigma_1 \) and \( \sigma_2 \) having the same sign. Note that the yield strength is reduced by the factor of safety \( n \).

2. von Mises or Distortion Energy Theory:

This states that failure occurs when the von Mises stress \( \sigma_e \) in the component being designed equals the von Mises stress \( \sigma_e \) in a uniaxial tensile test at the yield stress:

This gives:

\[
\sigma_e = \sqrt{2/2 \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{0.5}} = S_y/n
\]

In the plane stress case we have \( \sigma_3 = 0 \) and hence:

\[
\sigma_e = \left[ \sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2 \right]^{0.5} = S_y/n
\]

This is the most commonly used of the strength equations.

A third theory, the Maximum Normal Stress theory is similarly defined. It must NEVER be used for design with ductile materials. A modified version of this theory is sometimes used with brittle materials.

All three of these theories are shown on a plot the \( \sigma_1 \) versus \( \sigma_2 \) below: