HomeWork 3

Problem 1 20 points

Consider $Ax=b$ and let $A$ be the finite difference equations for solving $d^2T/dx^2 + q = 0$ for $N$ nodal points.

$A = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
-1 & 2 & -1 & 0 & 0 & 0 \\
0 & -1 & 2 & -1 & 0 & 0 \\
0 & 0 & -1 & 2 & -1 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\ldots & \ldots & \ldots & -1 & 2 & -1 \\
\ldots & \ldots & \ldots & \ldots & -1 & -1
\end{pmatrix}$

Solve the system of equations using values of $N=100$ and 1000 by the following methods

a. direct solution, getting the inverse during the process
b. Gaussian reduction using the Matlab command $x=A\backslash b$
c. Jacobi
d. Gauss-Seidel

Get the execution times using the cputime of Matlab and check the condition number of the matrices.

For (a and b) solve using $N=100:100:2000$ and see if the solution time is proportional to $N^2$? You may have to use $N$ greater than 2000 to get good results. Use a semi-log plot to check the results.

For (c) determine the maximum eigenvalue of $D^{-1}(L+U)$ directly using Matlab and by the power method.

For (c,d) plot the convergence as the iteration proceeds. Monitor the error and the residuals and comment on their behavior.

Since the first line of $AT = b$ is simply $T_1 = 0$, reduce the problem by eliminating the first equation and see if this makes any difference to the maximum eigenvalue of $D^{-1}(L+U)$ and the convergence.