Problem 1 5 points
In fitting the census data you chose different orders of fits. For the high orders, the condition numbers were so large that the fits were unacceptable and gave bad predictions.

Consider using the 7th order fit. The coefficients were determined by solving the linear equation set in the form $Ax = b$. Use the Matlab command $[U, S, V] = svd(A)$ and look at the singular values (the diagonal of $S$). When the ratio $S(i, i)/S(1, 1)$ is small you can expect that the values of $x$ will be very sensitive to small changes in $b$. Compare the values of $S$ for the different scaling methods.

Problem 2 20 points
As discussed in class yesterday we want to estimate the values of $V_0$ and $k$ in the model $V = V_0 \exp(-kt/m)$ where $V_0$ is the initial velocity and $kV$ represents the friction force.

1. Choose $V_0 = 20$ and $k/m = 0.1$. For times $t = [0 : 1 : 10]$ evaluate $V(t)$.
2. Define some noise by $\text{noise} = \text{randn}$ (randn generates random variables with a standard deviation of 1) and add some fraction of the noise to $V(t)$. The noise will range from $-4$ to $4$ so you need to add just a small fraction of the noise to $V(t)$, maybe about $1\%$ (this would make the standard deviation of the noise $=0.01$).
3. Determine the sensitivities of the model to $V_0$ and $k$ and following Eq. 3b in the notes Parameter-3 on the web, construct the $S$ matrix. The matrix will have two columns, one for $V_0$, the other for $k$. The vector of parameters will be $\theta = [V_0, k]^T$. Assume that the $\Sigma$ matrix is $\sigma^2$ times an Identity matrix. Then the $\sigma^2$ will cancel out of Eq. 3b.
4. Assume values for $V_0$ and $k$ and iterate using Eq. 3b until your estimates of $V_0$ and $k$ stop changing.
5. Evaluate the covariance matrix and find the standard deviation of your estimates and the correlation between the estimates.

Important Mathematical operations can only be done on dimensionless variables so you need to non-dimensionalize the model. You do this by defining $\overline{V} = V/V_r$, $\overline{t} = t/t_r$ and $\overline{k} = (k/m)/k_r$ where $V_r$, $t_r$, and $k_r$ are reference values. You could choose the numerical values to be $V_r = 20, t_r = 1, k_r = 0.1$. Then your sensitivities are of the form, $\partial M/\partial \overline{V}_0 = \partial(\overline{V})/\partial \overline{V}_0$. After you have iterated to convergence, the estimates of $V_0$ and $k$ will be equal to $\overline{V}_0 * V_r$. Remember that the ”measured data”, $D$ in Eq. 3b, must also be normalized.