1. Derive the exact solution of the Duffing’s equation for a hardening spring.

2. Consider the Rayleigh’s equation

\[ \ddot{x} + \varepsilon \left( \frac{1}{3} \dot{x}^3 - \dot{x} \right) + x = 0, \quad |\varepsilon| \ll 1 \]  

The equation is known to have a periodic solution in the form of a limit cycle. Use the Lindstedt-Poincaré approach to find a first approximation of the limit cycle.

3. Use the Lindstedt-Poincaré approach to find the first few terms in the expansion of the periodic solutions of

\[ \ddot{x} + x + \varepsilon x^2 = 0 \]  

Explain the presence of a constant term in the expansion.