ME 589, Vibration I
Homework 6

1. Consider the Van der Pol equation.

\[ \ddot{x} - \epsilon \left(1 - x^2 \right) \dot{x} + \omega^2 x = 0 \]  \hspace{1cm} (1)

(a) Integrate (1) numerically with \( \epsilon = 1 \) and initial conditions being \( x(0) = 0 \) and \( \dot{x}(0) = 0.25 \). Plot the trajectory on the phase plane to see if the system reaches a limit cycle as \( t \to \infty \). How does the shape of the limit cycle look like?

(b) Integrate (1) with \( \epsilon = 1 \) and initial conditions of your own. Does the trajectory land on the same limit cycle eventually?

(c) In the class, we use KBM method to predict the trajectory as

\[ a(t) = \frac{a_0}{\sqrt{\frac{\alpha^2}{4} + \left(1 - \frac{\alpha^2}{4}\right)e^{-ct}}} \]  \hspace{1cm} (2)

In the limit when \( t \to \infty, a \to 2 \). In other words, the limit cycle is a circle. Is this conclusion consistent with the limit cycle you found from the numerical integration? If not, please explain why.

2. Consider the Duffing’s equation with dry friction. The normalized equation of motion is

\[ \ddot{u} + u = \epsilon \left(f - \alpha u^3 \right) + 2\epsilon k \cos \Omega t \]  \hspace{1cm} (3)

where \( f \) models the dry friction with

\[ f(u, \dot{u}) = \begin{cases} -1, & \text{if } \dot{u} > 0 \\ 1, & \text{if } \dot{u} < 0 \end{cases} \]  \hspace{1cm} (4)

In addition, consider a hardening spring with \( \alpha > 0 \). Determine the steady-state solutions of primary resonances of this system, and obtain the frequency-response equation. Determine the stability of the steady-state solution. Plot the amplitude \( a \) and the phase \( \gamma \) as functions of the detuning \( \sigma \) for \( k = 2/\pi \) and \( k > 2/\pi \). What is the significance of \( k = 2/\pi \)? Will there be a jump phenomenon?

3. Consider the superharmonic resonance of the forced Duffing’s equation discussed in our lectures. Verify the stability of the periodic solutions.