1) **Adhesive Contact and Critical Contact Radius**

Determine the adhesion force, \( F_{\text{adh}} \), and the corresponding critical contact radius, \( a_c \), between two spheres using the JKR model.

2) **Adhesion of Elastic Spheres – JKR vs. Bradley: The Tabor Coefficient**

(a) Below you find the sketch of the LJ force for two spheres. Does the LJ force contain any elastic components in the limit of reversibility?

   **If yes**, show the critical distance at which the elastic component becomes recognizable in the force curve,

   **if no**, add schematically to the plot below a extended LJ force curve which is considering elastic deformation of the spheres but is still reversible.

![Sketch of LJ force](image)

(b) Sketch paired force curves (axis chosen as in (a)) which illustrate the differences between:

   (i) infinitely stiff spheres with **larger or smaller** molecular distance (\( \sigma_1 > \sigma_2 \)),

   (ii) elastic spheres (with reversible force curves) with **larger or smaller** Young's modulus (\( E_1 > E_2 \)), and

   (iii) **reversible or irreversible** force curves (\( F_{\text{rev}}, F_{\text{irrev}} \)). What can be used as an indicator if the force curve is reversible or irreversible?

(c) Sketch (into the same plot) the course of the force during a dissipative approach and retraction. Add directional arrows. Note: The measuring system is still considered to be infinitely stiff.

Label the plot and discuss the details during approach and retraction (i.e., instabilities (where? why?), energy dissipation, JKR and Bradley branch).
(d) To enhance dissipation in a zero compliance measuring system would you rather choose a large or a small effective radius $R^*$. Why?

(e) Assuming your measuring system has a non-zero compliance (i.e., involves a spring with finite spring constant), and the Tabor coefficient is significantly larger than 1, sketch as in (c), the course of the force approach and retraction curve.

(f) Which spring constant and radius of curvature would you choose for AFM measurements to probe as close as possible to the sample instabilities as defined by (ABCD).

- a very stiff cantilever with a large tip radius,
- a very soft cantilever with a small tip radius
- a very soft cantilever with a large tip radius
- a very stiff cantilever with a small tip radius

3) **Discussions between Tribologists**

After David Tabor's talk at the NATO Conference on Tribology in Braunlage (Germany) in 1991, Jacob Israelachvili posed the following question: "I would like to discuss a point raised at the end of your lecture when you suggested that there is a reversible energy change (surface energy) if surfaces are separated slowly enough. Can that really be reversible?"

How would you answer the question? Elaborate.
1) **Adhesive Contact and Critical Contact Radius**

Determine the adhesion force, $F_{adh}$, and the corresponding critical contact radius, $a_c$, between two spheres using the JKR model.

\[ JKR: \quad a^3 = \frac{3R}{4E^*} \left( L + 3\frac{\mu}{R} R + \sqrt{6\frac{\mu}{R} R L + (3\frac{\mu}{R} R)^2} \right) \]

\[ L_c = -\frac{F_{adh}}{\pi} \]

(i) generate from JKR eq. an implicit function

\[ F(L(a), a) = 0 \]

\[ \left( L - \frac{4E^*a^3}{3R} \right)^2 - 16\frac{\mu}{R} E^* a^3 = 0 \]

(ii) use the implicit function theorem to determine

\[ \frac{dL}{da} = -\frac{\partial F/\partial a}{\partial F/\partial L} \]

\[ \implies a_c^3 = \frac{9\frac{\mu}{R} R^2}{4E^*} \]

and set \[ \frac{dL}{da} = 0 \]

\[ L_c = -\frac{F_{adh}}{\pi} = -3\frac{\mu}{R} R \]
2) **Adhesion of Elastic Spheres – JKR vs. Bradley: The Tabor Coefficient**

(a) No. The LJ-force between two spheres is consistent with Bradley's model which assumes infinitely stiff spheres. Find below sketched the "Derjaguin-Bradley" force curve predicted by Greenwood. Note that the force minimum of "Derjaguin-Bradley" is shifted towards larger separation distance the larger the effect of the elastic deformation is. The contact distance \( D_0 \) is increased with the increasing effect of the elastic deformation.

The *Tabor Coefficient* is used as an indicator. For a Tabor coefficient smaller than 1 the force curve is reversible for an infinitely stiff measuring system. For a Tabor coefficient larger than 1 the force curve is irreversible for an infinitely stiff measuring system. The Tabor coefficient is a measure of the ratio of surface energy and the range of surface forces.
The force curve consists of a lower branch at large separation distance and an upper branch at closer separation distance. Transitions from one to the other branch occur at the two instability points "A" and "C". At the instability the gradient of the force becomes infinite.

"A" is the instability point during the approach. At "A" the two surfaces deform suddenly towards each other and form an adhesive interfacial junction at "B". The location of the overall center of mass of the sphere is not affected by this instability.

"C" is the instability point during the retraction. At "C" the adhesive junction is suddenly lost and the force jumps from the upper branch to the lower LJ branch. Because we assumed an infinitely stiff system the location of the overall center of mass of the sphere is again not affected.

The area described by (ABCD) corresponds to the energy dissipated during the approach-retraction cycle.

(d) A small effective radius $R^*$ is enhancing dissipation, i.e., is increasing the area (ABCD).

The reason is the ratio between the contact periphery ($L=2\pi a$) and the contact area ($A=\pi a^2$): $L/A=2/a$. For smaller spheres, $L/A>>1$, the contact periphery becomes more important. The contact periphery is the reason for the sample instability. The radius is related to the contact area by $R=a^2/\delta$. Hence dissipation becomes larger for a small effective radius.
The slopes of the dashed lines correspond to the stiffness of the measuring system (for instance the cantilever spring constant). The system shows instabilities during approach and retraction at "A₀" and "C₀" which are always located before the instabilities of the sample materials (i.e., "A" and "C").

(f) A stiff cantilever with a large tip radius of curvature is enhancing the sample's instability and reducing the cantilever instability.
3) **Discussions between Tribologists**

It takes a great man that admits that he is wrong!

**Tabor's Response**

"Before I attempt to express my main impressions of this Conference I feel I must interpose a few words as a penitent and apologize for the mistake I made at the end of my original presentation. On that occasion I presented a model which I thought demonstrated that in pulling two surfaces apart the process was reversible and only the surface energy was involved. I now realize that this is not so and that as the bonds go "pop" across the interface elastic energy is lost in the bodies. I do not know if this is true in peeling. But my original model involving the separation across a plane appears to be wrong. ....

As a practicing scientist I recognize three levels of achievement.

- The first and best is to get it right and to have the results accepted by others.

- The second is to get it wrong or to be controversial and to evoke discussion which finally resolves the issue.

- The third and least satisfactory is to have your work ignored. My involvement in surface energy is in the second category."

*See next page for selected student responses:*
Problem 3 continued: Selected student responses:

Pro

"...If no energy is dissipated as heat....then reversible energy change is possible. ...The thermodynamic work of adhesion (W=γ₁+γ₂ - γ₁₂ ; γ is the surface free energy of the two separated surface, i=1,2, and γ₁₂ is the interfacial free energy) is the work required to separate the two initially contacting surfaces. ...If the amount of energy necessary to separate the two surfaces is equal to the gained energy during formation of the interface (γ₁₂) the process is reversible."

"...The process can be reversible if the contact is non-conforming elastic and the displacement is elastic."

Contra

"...A reversible energy change implies that there is no loss of energy; the process is elastic and conserves energy. However, given two surfaces that produce an adhesion interface, the amount of force necessary to separate these two surfaces is greater than the amount of force necessary to create the interface. As a result, energy does not appear to be conserved...

"...A reversible process is defined as a “quasi-static process in which interacting systems are .... infinitesimally removed from mutual equilibrium (Lecture Noses in Thermodyn. J.C. Berg (1997)”. Quasi-static processes involve systems in internal partial equilibrium w.r.t. thermal, mechanical and diffusional processes. By these definitions, adhesion is irreversible... “