7.A. Semi-insulating GaAs.
All practical semiconducting materials will have some number of unavoidable impurities that give rise to states within $kT$ of the band edges (so-called “shallow” donors and acceptors). Let these inherent impurity concentrations be $n_D$ donors and $n_A$ acceptors, with $n_D < n_A$. Without intervention, the number of free carriers in the system will be $\approx |n_A - n_D|$. To greatly reduce the number of free electrons and holes, one can deliberately add a concentration $n_m$ of “deep” donors, with an energy level near the center of the band-gap, so that it takes energy $E_g/2$ to remove the electron from the deep donor and put it in the conduction band. Show that if $n_m > n_A - n_D$, charge neutrality requires that the deep donor level be partially occupied. Hence, show that the doping lowers the carrier density from $n_A n_D$ to a value near the intrinsic carrier concentration $n_i$. Assume $kT$ is much less than the band gap.

7.B. Intrinsic vs. Extrinsic Conductivity
Conductivity $\sigma$ relates the current density $J$ to an applied electric field $E$ as $J = \sigma E$. Since the current density depends on the carrier density, charge and velocity as $J = n q v$, it is convenient to separate out the role of the number and charge of the carriers from that of the intrinsic scattering and density of states in the material, and define the sample mobility $\mu = \sigma / n e$, via $v = \mu E$, relating the drift velocity of an individual carrier to the electric field. The mobility is characteristic of the material, and not the number of carriers that might be induced either externally or internally (although large numbers of carriers can reduce the mobility through scattering effects).

Consider a germanium crystal with a dopant concentration of 1 part per million As atoms (0.0001%), or $N_D = 10^4 n_{Ge} = 10^4 * 4.4 \times 10^{22} \text{cm}^{-3} = 4.4 \times 10^{16} \text{cm}^{-3}$. The binding energy for As donors is 12.7 meV in Ge. The density of states effective masses are $m_e = 0.55 m_e$ and $m_h = 0.38 m_e$. At 300 K and $4 \times 10^{16} \text{cm}^{-3}$ doping levels, the mobilities are $\mu_n \sim 3000 \text{cm}^2/\text{V-s}$ and $\mu_p \sim 1000 \text{cm}^2/\text{V-s}$.

a) Show that at room temperature, essentially all the donors are ionized. How will the impurity contribution to the conductivity vary as the temperature increases above room temperature?
b) At about what temperature will the intrinsic conductivity (from thermally excited electron-hole pairs) equal the impurity-induced conductivity? If you wish, you may simplify the expression by assuming equal mobilities and effective masses for electrons and holes.
c) Sketch the temperature dependence of the conductivity of the doped Ge sample, labeling the various contributions.

7.C. Degenerate semiconductor (adapted from Sturge 13.5)
In class, we assumed the doping levels were much less than the quantum concentration, $n_D << n_i$ and $n_A << n_i$ so that we could treat the excited carriers as an ideal gas. However, the quantum concentrations depend on temperature, and approach zero as $T$ goes to zero, so at some point, this condition will not be satisfied.
a) Make a rough sketch of density of states vs. energy for a semiconductor near the band gap, and show the location of the Fermi level at $T = 0$ K in a semiconductor that is (i) strongly n-type and (ii) strongly p-type. Show that if $n_e >> n_h$, the electrons form a degenerate Fermi gas and the chemical potential is given by $\mu = E_c + \frac{\hbar^2}{2m_e} \left( \frac{3\pi^2 n_e}{2} \right)^{2/3}$ (ABOVE the CBM).
b) In practice, doping levels are often used that are greater than the quantum concentrations even at room temperature. Estimate the minimum doping level required for the above solution to be valid in GaAs at room temperature, given that $m^* = 0.07 m_e$. 

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**Homework 7 – Semiconductor Statistics.**
OPTIONAL PROBLEM
This problem doesn’t really involve statistical mechanics once you have generated the charge depletion region at a \(pn\) junction. It is a nice exercise in E&M that the interested student may want to work through to understand the size of the voltage drop in a \(pn\)-junction. The problem is thus optional, and won’t be graded.

7.D PN Junction Fields
Assume that a \(p-n\) junction is characterized by two sharply defined charge regions of charge density \(\rho\) as shown, where \(N_a\) and \(N_d\) are the density of acceptors and donors, respectively, and the space charge depletion widths are \(W_a\) and \(W_d\). In this central region, the free carrier densities are orders of magnitude smaller than in the bulk of the device since the electrons and hole recombine.

a) For \(x<0\), you can write the Poisson equation (MKS units) as
\[
\frac{\partial^2 V}{\partial x^2} = \frac{qN_a}{\varepsilon}
\]
where \(\varepsilon = \) dielectric constant \((=\kappa_0)\). We are using MKS so the answers will be in Volts, and not stat-volts or esu. Integrate this equation and a similar one for \(x>0\) to show that the barrier voltage is given by:
\[
V_b = V_1 + V_2 = \frac{q}{2\varepsilon} \left( N_a W_a^2 + N_d W_d^2 \right).
\]
Note that from charge neutrality, \(W_d N_d = W_a N_a\), so that it may also be written as:
\[
V_b = \frac{q W_d^2 N_d}{2\varepsilon} \left( 1 + \frac{N_a}{N_d} \right).
\]

b) Sketch the potential as a function of \(x\) through the junction. Justify your linear, quadratic, or whatever you find, dependence of the potential, as well as which side is positive or negative. Show that the width of depletion region \(i\) \((j\) is the other region) is given by:
\[
W_i = \sqrt{\frac{2eV_b}{qN_i \left( 1 + \frac{N_j}{N_i} \right)}}
\]

c) Show that in the case where \(N_a \ll N_d\), the width of the depletion region to a good approximation may be written as \(W_{tot} = \sqrt{\frac{2eV_b}{\sigma}}\), where \(\sigma = \) conductivity and \(\mu = \) mobility.

d) The junction capacitance is defined by \(C_j = \frac{dQ}{dV_b} = \frac{dQ}{dW} \frac{dW}{dV_b}\). In the practical case for which one side of the junction is very lightly doped while the other is heavily doped \((N_a \ll N_d)\), show the capacitance may be written:
\[
C_j = \frac{\varepsilon \sigma}{2\mu V_b}
\]

e) Silicon of resistivity \(\rho = 1 \text{ \Omega-cm}\) at room temperature has carrier density \(\sim 10^{15} \text{ cm}^{-3}\), mobility \(\sim 10^3 \text{ cm}^2/\text{V-sec}\), \(\sigma = 1 \text{ mho/cm}\), \(\varepsilon = 11.9\varepsilon_0\), \(V_b = 0.7 \text{ V}\). Find the width of the depletion region and the junction capacitance in the case \(N_a \ll N_d\).