Physics 228, Winter 2014

SECOND MIDTERM EXAMINATION (Thursday, March 6th, 2014)

Name _______________________________ St. ID No. _____________
(print) Last First

(1) ___________; (2) ___________; (3) ___________; (4) ___________

Total: ___________ /50

Exam procedures PLEASE READ.

• Write your name and student number above, and also write your name ON THE TOP
  OF EACH ODD-NUMBERED PAGE.

• Please sit away from other students.

• If you have a question about the exam, please ask.

• This is a closed book exam. A list of relevant formulae is given on the equations sheet.
  If you think a needed equation is not provided, please ask. No calculators are allowed.

• Note that questions appear on both sides of the sheet (including the back of this page).

• Write your answers on the exam. I have tried to leave ample space, but if your need
  more, use additional paper and be sure to write “228 Midterm” and your name on the
  top of each additional page.

• Please show your work, explaining the logic of your calculation. A correct answer
  alone with no explanation will usually not get full credit, unless you are asked to
  “write down” or “state” an answer.

• Timing. Each point on the test corresponds to 1 minute of real time. Thus, for
  example, you should spend about 10 minutes (or less) on a 10 point question.
1. **[10 pts total] Lagrangian mechanics.** A particle of mass $m$ moves in three dimensions and is described using cylindrical coordinates $\{r, \theta, z\}$. It is subject to a potential $V = \alpha r^4 + \beta z^2$. Determine the equations of motion, i.e. find the differential equations that must be solved to find $r(t)$, $\theta(t)$ and $z(t)$. You do not need to solve these equations. However, you should give an expression for one conserved quantity other than the total energy, and state its physical interpretation. (A conserved quantity is a quantity that is independent of time.)
2. [10 pts] Gamma and Beta functions.

(a) It is known that $\Gamma(1.5) = \sqrt{\pi}/2$. Find the value of $\Gamma(-1.5)$.

(b) Express the following integral in terms of Gamma and/or Beta functions:

$$I = \int_0^\infty \sqrt{x} e^{-x^3} \, dx.$$ 

You do not need to evaluate it numerically.
3. [11 pts] Legendre Series. Determine the coefficients $c_n$ in the Legendre series

$$P'_5(x) = \frac{dP_5(x)}{dx} = \sum_{n=0}^{\infty} c_n P_n(x).$$

Hints: First use general arguments to determine which coefficients are non-zero, then calculate the non-zero coefficients, using integration by parts. Also, you do not need to know the explicit form of $P_5(x)$ to do this problem.
4. [19 pts total] PDEs and separation of variables. We consider here the temperature distribution inside a long, narrow rod of length $L$ with insulated sides, which we can solve by treating it as a 1-D diffusion problem. If $x$ is the coordinate along the rod (running from 0 to $L$), then the temperature $T(x,t)$ satisfies

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha^2} \frac{\partial T}{\partial t},$$

with $\alpha$ a constant. For $t < 0$, the slab is in steady-state equilibrium, with the $x = 0$ end held at $0^\circ$ while the end at $x = L$ held at temperature $T_0 > 0^\circ$. At $t = 0$, and henceforward, both ends are held at $0^\circ$.

(a) [4 pts] Separate variables $T = X(x)\tau(t)$ and write down the differential equations satisfied by the $x$ and $t$-dependent functions.

(b) [5 pts] Solve for $T(x,t)$ for $t < 0$. 
(c) [6 pts] Write down the general form of the solution $T(x, t)$ for $t > 0$, incorporating the boundary conditions in $x$. This form will contain undetermined constants.
(d) [4 pts] Derive a formula for the undetermined constants in your expression. You do not need to explicitly evaluate any integrals.