Obtaining Laurent Series & residues using *Mathematica*

Laurent Series example discussed in Boas and in class

```
In[343]= Clear[ff]
In[344]= ff[z_] = 12 / (z (2 - z) (1 + z))
Out[344]= 12
       (2 - z) z (1 + z)
```

**Inner region R1**

*Mathematica* command Series[] automatically gives Laurent series. `{z,0,3}` means: expand in z, about z=0, giving up to z^3 term.

```
In[345]= Series[ff[z], {z, 0, 3}]
Out[345]= 6 - 3 + 9 z - 15 z^2 + 33 z^3 + O[z]^4
```

**Outermost region R3**

Here we expand about z=Infinity, and *Mathematica* automatically does the series in powers of 1/z

```
In[347]= Series[ff[z], {z, Infinity, 6}]
        z^3 z^4 z^5 z^6
```

**Middle region R2**

Partial fraction function:

```
In[372]= Clear[ff1, ff2]
In[373]= ff1[z_] = 4 / (z (1 + z)) ; ff2[z_] = 4 / (z (2 - z));
    Check that sum agrees with original function
In[374]= FullSimplify[ff1[z] + ff2[z] - ff[z]]
Out[374]= 0
In[379]= Normal[Series[ff1[z], {z, Infinity, 4}]]
Out[379]= 4 z - 4 z^2 + 4 z^3
       z^4 + z^3 + z^2
In[380]= Normal[Series[ff2[z], {z, 0, 3}]]
Out[380]= 2 + z + z^2 + z^3
       2 4 8
```
In[381]:= \[textbf{result} = ff1expand + ff2expand\]

Out[381]= 
\[
1 + \frac{4}{z^4} - \frac{4}{z^3} + \frac{4}{z^2} + \frac{2}{z} + \frac{2}{4} + \frac{z^2}{4} + \frac{z^3}{8}
\]

**Residues**

This is a built-in \textit{Mathematica} command. Here is the example of the function discussed above.

In[382]:= \textbf{Residue}[ff[z], \{z, 0\}]

Out[382]= 6

In[384]:= \textbf{Residue}[ff[z], \{z, -1\}]

Out[384]= -4

In[385]:= \textbf{Residue}[ff[z], \{z, 2\}]

Out[385]= -2