If $I = \int F(x, y, y')dx$ is stationary, with $y(x)$ fixed at the endpoints, then

$$\frac{d}{dx} \frac{\partial F}{\partial y'} - \frac{\partial F}{\partial y} = 0.$$ 

If $F$ is independent of $x$ then $F - y'\frac{\partial F}{\partial y'}$ is a constant.

The aim in this problem is to find $y(x)$ with $y(0) = 2$ and $y(1) = 0$ such that

$$I[y] = \int_0^1 (1 + 2y')^2dx$$

is minimized.

1. (10 pts) Write down the Euler equation satisfied by $y(x)$ and determine its first integral (i.e. integrate it once).
2. (10 pts) Do the second integral to determine the general form for $y(x)$, and then use the boundary conditions to determine the specific solution. How do we know that this solution minimizes $I[y]$, rather, say, than maximizing it?