Exam 1 solutions

1. A cube of metal has a mass of 0.5 kg. It measures 2.1 cm on a side. Calculate its density.
   A. 40,500 kg/m³
   B. 7000
   C. 11,000
   D. 54,000
   E. 6150
   Answer: D $V_{\text{cube}} = L^3$, $r = \frac{M}{V} = \frac{0.5\text{kg}}{(0.021\text{m})^3} = 54,000\text{ kg/m}^3$

2. You pour water (density 1000 kg/m³) into a cylinder till it reaches a depth of 10 cm. Then you pour in 30 cm of olive oil (density 800 kg/m³) on top of the water. Given the pressure is 1 atm at the open top of the cylinder then what is the absolute P at the bottom (in kilo-Pascals)?
   A. 97.9
   B. 102.7
   C. 104.6
   D. 101.3
   E. 134.8
   Answer: C $P = \rho g h$ for each fluid, + atm
   $P = (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.10\text{m}) + (800 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.30\text{m}) + 101.3 \text{ kPa} = 104.6\text{kPa}$

3. A block of iron and a block of wood have the same volume. The wood floats, partially out of the water, but the iron sinks. On which block is the buoyant force larger?
   A. Wood
   B. Iron
   C. Same
   Answer: B $BF = \text{weight of water displaced}, \text{blocks have same V, but ALL of iron is underwater, only part of wood}$

For questions 4 and 5: When a certain hollow glass sphere floats freely on a freshwater lake, half of the sphere is below the surface of the water. The mass of the sphere is $m$. Assume the water is incompressible.
A (massless) string is attached to the bottom of the sphere and connected to a rock at the bottom of the lake. The string holds the sphere entirely below the surface of the lake.

4. Does the magnitude of the buoyant force on the sphere depend on how far the top of the sphere is beneath the surface of the water?
   A. Yes
   B. No
   Answer: B $BF = \text{weight of water displaced}, \text{water density is same at all depths}$
5. (6 pts) The magnitude of the force that the string exerts on the sphere is:
   A. 0
   B. 2mg
   C. 3mg
   D. mg /2
   E. mg
   Answer: E
   \[
   \text{floating : } mg \rightarrow \rho_w g \left( \frac{V_B}{2} \right) = \rho_B g V_B \rightarrow \rho_B = \frac{\rho_w}{2}
   \]
   \[
   \text{submerged : } mg + F = B \rightarrow F = B - mg = \rho_w g V_B - \rho_B g V_B
   \]
   \[
   F = \left( 2 \rho_B - \rho_B \right) g V_B = \rho_B g V_B = mg
   \]

6. Find the pressure difference on an airplane wing if the air flows over the upper wing at 150 m/s and along the bottom surface with a speed of 120 m/s? Thickness of wing is negligible.
   A. 3127 Pa
   B. 5225
   C. 2838
   D. 1841
   E. 8640
   Answer: B
   \[
   P + \frac{1}{2} \rho v^2 + \rho gh = \text{const}, \Delta h \text{ is negligible, so}
   \]
   \[
   \Delta P = \frac{1}{2} \rho \left( v_{\text{top}}^2 - v_{\text{bottom}}^2 \right) = \frac{1}{2} \left( 1.23 \frac{\text{kg}}{\text{m}^3} \right) \left[ \left( 150 \frac{\text{m}}{\text{s}} \right)^2 - \left( 120 \frac{\text{m}}{\text{s}} \right)^2 \right] = 5225 \text{ Pa}
   \]

For 7 – 10: Water with negligible viscosity flows in a horizontal pipe, with cross-sectional area 24.0 cm$^2$, and the speed of the water is 2.80 m/s. Then the pipe narrows, and the cross-sectional area becomes 16.0 cm$^2$.

7. The volume flow rate in the pipe is
   A. Larger in the wide part
   B. Larger in the narrow part
   C. The same in both parts.
   Ans: C (mass conservation)

8. What is the speed of water flow in the narrow part?
   A) 1.86 m/s
   B) 2.80 m/s
   C) 4.20 m/s
   D) 6.30 m/s
   E) None of the above
   Answer: C
   \[
   A_1 V_1 = A_2 V_2 \rightarrow V_2 = V_1 \left( \frac{A_1}{A_2} \right) = \left( 2.8 \frac{\text{m}}{\text{s}} \right) \left( \frac{24 \text{cm}^2}{16 \text{cm}^2} \right) = 4.2 \frac{\text{m}}{\text{s}}
   \]
9. In which part of the pipe is the pressure higher?
A. Wider section  
B. Narrower section  
C. No difference  
Ans: A

\[ P + \frac{1}{2} \rho v^2 = \text{const} \rightarrow P_2 + \frac{1}{2} \rho v_2^2 = P_1 + \frac{1}{2} \rho v_1^2 \]

\[ P_2 = P_1 + \frac{1}{2} \rho \left(v_1^2 - v_2^2\right), \quad v_2 > v_1 \rightarrow P_2 < P_1 \]

10. If the fluid velocities in the wide and narrow pipes are \( v_1 \) and \( v_2 \) respectively, the magnitude of the difference in pressure between pipes is proportional to
A. \(|v_2 - v_1|\)  
B. \(|\frac{v_2}{v_1}|\)  
C. \(|\frac{v_1}{v_2}|\)  
D. \(|v_2^2 - v_1^2|\)  
E. 0  
Ans: D (see question 9 above)

11. A viscous fluid flows through a horizontal tube at an average flow rate of 16 cm\(^3\)/s. If the tube diameter was half as large, but the length and pressure difference across the ends of the tube remained the same, what would be the flow rate (in cm\(^3\)/s)?
A. 1  
B. 2  
C. 4  
D. 8  
E. 16  
Ans: A

\[
I = \frac{\Delta P \pi r^4}{8\eta L} \rightarrow \frac{I_2}{I_1} = \left(\frac{r_2}{r_1}\right)^4 = \left(\frac{1}{2}\right)^4 = 1/16
\]

12. (6 pts) What is the net power that a person with surface area of 1.20 m\(^2\) radiates, if his emissivity is 0.895, his skin temperature is 300 K, and he is in a room that is at a temperature of 290 K? The Stefan-Boltzmann constant is 5.67 x 10\(^{-8}\) W/(m\(^2\)·K\(^4\)).
A 60.3 W  
B 62.6 W  
C 65.7 W  
D 68.4 W  
E 64.8 W  
Answer: B
\[ P_{\text{rad}} = \varepsilon A \sigma \left( T^4 - T_0^4 \right) = (0.895)(1.2m^2) \left( 5.67 \times 10^{-8} \text{ W/(m}^2\text{K}^4) \right) \left( [300K]^4 - [290K]^4 \right) = 0.895 \times 1.2 \times 5.67 \times 10^{-8} \times \left( 300^4 - 290^4 \right) \text{ W} \]
\[ P_{\text{rad}} = 62.6W \]

13. (8 pts) The sun delivers heat energy (solar power) at the surface of the earth at the rate of about 1 kW/m².
Sunlight heats a pan of water whose area is 1 m² and whose depth is 0.1 m. (Assume all solar energy goes into heat in the water; ignore energy losses to surroundings and reflection.)
What is the approximate temperature rise (in °C) of the water after one hour (3600 sec)?
A. 3
B. 6
C. 9
D. 12
E. 15
Answer: C

\[
1kW / m^2 = 1kJ / s / m^2 \rightarrow 3600kJ / hr / m^2 \rightarrow Q = 3600J
\]
\[ V = 0.1m^3 \rightarrow m = 100kg \]
\[ Q = mc\Delta T \rightarrow \Delta T = Q / mc \]
\[ \Delta T = 3600kJ / (100kg) \left( 4186J / kg / K \right) = 3600 / 4186 \approx 9K \]

14. (8 pts) The specific heat of aluminum is 900 J/kg/K. A piece of aluminum with mass 0.1 kg and temperature 50 °C is placed in 1 kg of water whose temperature is 20 °C, in an insulated container. The equilibrium temperature is approximately (°C):
A. 21
B. 28
C. 36
D. 42
E. 32
Ans: A

\[
\Delta Q_{Al} = -\Delta Q_w \rightarrow m_{Al}c_{Al}\Delta T_{Al} = -m_wc_w\Delta T_w
\]
\[
(0.1 \text{ kg})\left( 900J / kg / K \right)(T_f - 50C) = (1 \text{ kg})\left( 4186J / kg / K \right)(20C - T_f)
\]
\[
(90J / K)T_f - 4500J = 83720J - (4186J / K)T_f
\]
\[
(4276J / K)T_f = 88220J \rightarrow T_f = 20.6C \approx 21C
\]
15. An ideal gas has a temperature of 400 K, a pressure of 2 atm and fills a volume of 20 liters. Find the number of moles of this gas.
   A. 2.4 mols
   B. 1.62
   C. 1.43
   D. 1.22
   E. 1.07
   Answer: D

\[ PV = nRT \rightarrow n = \frac{PV}{RT} = \frac{2 \text{ atm} \left(101.3 \text{ kPa/atm}\right) \cdot 20 \text{ l} \left(10^{-3} \text{ m}^3/\text{l}\right)}{\left(8.31 \text{ J/mol} \cdot \text{K}\right) \cdot 400 \text{ K}} = 1.22 \text{ mol} \]

16. (6 pts) A cylindrical flask of cross-sectional area \( A \) has a gastight piston that is free to slide up and down. Inside the flask is an ideal gas. Initially the pressure applied by the piston to the gas is \( 200 \times 10^3 \text{ Pa} \), and the piston is stationary at a height of 0.2 m above the base of the flask. Additional mass is now put onto the piston, and the gas pressure rises to \( 250 \times 10^3 \text{ Pa} \). The temperature is constant at 300 K. What is the new height of the piston (in m)?
   A. 0.01
   B. 0.02
   C. 0.04
   D. 0.08
   E. 0.16
   Ans: E

\[ V_2 = h_2 A, \quad V_1 = \left(0.2 m\right) A \]

\[ P_1 V_1 = P_2 V_2 \rightarrow P_2 h_2 A = P_1 \left(0.2 m\right) A \rightarrow h_2 = \frac{P_1}{P_2} \left(0.2 m\right) = 0.16 \text{ m} \]

17. A rod of copper has a cross section of \( 0.0002 \text{ m}^2 \) and a length of \( 1 \text{ m} \). One end is at 100 \(^\circ\) C, the other at 0 \(^\circ\) C. Find the heat flow in the rod.
   (a) 3.96 Watts
   (b) 2.63
   (c) 2.12
   (d) 1.74
   (e) 7.90
   Answer: E

\[ \text{Copper conductivity } k_{Cu} = 395 \text{ (W/m} \cdot \text{K)} \]

\[ \frac{Q}{t} = k_{Cu} A \Delta T/L = 395 \text{ (W/m} \cdot \text{K)} (0.0002 \text{ m}^2)(100 \text{K})/1 \text{m} = 7.9 \text{ J/s} \]
18. (6 pts) A balloon containing 2.0 m$^3$ of hydrogen gas rises from a location at which the temperature is 22°C and the pressure is 101 kPa to a location where the temperature is -39°C and the pressure is 20 kPa. If the balloon is free to expand so that the pressure of the gas inside is equal to the ambient pressure, what is the new volume of the balloon, in m$^3$?

A  4.0
B  6.0
C  8.0
D  10
E  12

Answer:  C

NOTE: in the problem statement, the minus sign was not clearly attached to 39°C, so you may have read T$_2$ as +39°C. Result would be 10.6 m$^3$. SO, answer D is also accepted as correct.

$$PV = NkT \quad \text{with} \quad N = \text{const}$$

$$\frac{PV}{T} = \text{const} \rightarrow V = \left( \text{const} \right) \frac{T}{P} \rightarrow \frac{V_2}{V_1} = \frac{T_2}{T_1} \frac{P_1}{P_2} = \frac{234 K}{20 kPa} \frac{101 kPa}{295 K} = 4.0$$

$$V_2 = 4V_1 = 8 m^3$$