Constants you may find useful

\[ G = 6.67 \times 10^{-11} \text{Nm}^2/\text{kg}^2 \]
\[ M_{\text{Earth}} = 5.97 \times 10^{24} \text{kg} \]
\[ M_{\text{Sun}} = 1.99 \times 10^{30} \text{kg} \]
\[ r_{\text{Earth}} = 6.38 \times 10^3 \text{km} \]
\[ r_{\text{Sun}} = 6.96 \times 10^4 \text{km} \]
\[ d_{\text{Sun-Earth}} = 149.6 \times 10^6 \text{km} \]
\[ g = 9.8 \text{m/s}^2 \]
\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{C}^2/(\text{Nm}^2) \]
\[ \mu_0 = 4\pi \times 10^{-7} \text{Tm/A} \]
\[ e = 1.6 \times 10^{-19} \text{C} \]
\[ h = 6.626 \times 10^{-34} \text{Jsec} \]
\[ m_e = 9.11 \times 10^{-31} \text{kg} \]
\[ m_p = 1.6726 \times 10^{-27} \text{kg} \]
\[ c = 3 \times 10^8 \text{m/s} \]
Part I. [37 Points] In the Bohr model for the hydrogen atom, the distance between the electron and the proton in the ground state is one Bohr radius, 0.0529 nm.

1. [5 pts] Find the force on the proton in Newton's when the atom is in the ground state.
   - A. $1.0 \times 10^{-27}$ N
   - B. $4.4 \times 10^{-18}$ N
   - C. $8.2 \times 10^{-8}$ N
   - D. $1.0 \times 10^{-6}$ N
   - E. $5.1 \times 10^{11}$ N

   \[ F = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r^2} = 9 \times 10^9 \left(\frac{1.6 \times 10^{-19}}{0.0529 \times 10^{-9}}\right)^2 \]
   \[ r = 8.233 \times 10^{-8} \text{ N} \]

2. [4 pts] SI units are often not convenient for atomic problems. Find the force in number 1 in units appropriate to atomic physics, eV/nm.
   - A. $6.4 \times 10^{-18}$ eV/nm
   - B. $2.7 \times 10^{-8}$ eV/nm
   - C. $5.1 \times 10^{2}$ eV/nm
   - D. $6.5 \times 10^{3}$ eV/nm
   - E. $3.2 \times 10^{21}$ eV/nm

   \[ F = \frac{1}{1.6 \times 10^{-19}} \cdot \frac{e^2}{r^2} \]
   \[ r = 514. \text{ eV/nm} \]

3. [5 pts] Find the magnitude of the momentum of the electron in the hydrogen atom ground state in SI units.
   - A. $1.5 \times 10^{-29}$ kg/m/s
   - B. $2.0 \times 10^{-24}$ kg/m/s
   - C. $7.1 \times 10^{-24}$ kg/m/s
   - D. $4.4 \times 10^{-18}$ kg/m/s
   - E. $5.0 \times 10^{-15}$ kg/m/s

   \[ p = \sqrt{\frac{mV^2}{r}} \]
   \[ r = \sqrt{\frac{mF}{V}} \]
   \[ p = mV = \sqrt{rmF} = \sqrt{(0.0529 \times 10^{-9}) (9.11 \times 10^{-31}) (8.23 \times 10^{-8})} \]
   \[ p = 514 \text{ eV/nm} \]

4. [4 pts] The unit for momentum in units appropriate to atomic physics is eV/c, i.e. eV (the unit for energy) divided by the speed of light. Find the momentum of part 3 in units of eV/c.
   - A. $2.7 \times 10^{-2}$ eV/c
   - B. $3.7 \times 10^{3}$ eV/c
   - C. $1.3 \times 10^{4}$ eV/c
   - D. $8.2 \times 10^{9}$ eV/c
   - E. $9.3 \times 10^{12}$ eV/c

   \[ p = \frac{p}{1.6 \times 10^{-19}} \]
   \[ p = 3712.5 \text{ eV/c} \]

5. [5 pts] The H atom in its ground state is excited to what state by the absorption of a photon with wavelength 103 nm (infrared)?
   
   - A. N=1
   - B. N=2
   - C. N=3
   - D. N=4
   - E. N=5

   This is UV, not infrared (this does not affect the problem, however)

   Well, I hate rounding. An equally valid answer for this problem is that since it doesn't hit the N=3 energy exactly, it will not be absorbed at all. I'll give credit for either A or C.
Consider the *emission* spectra at right for the H atom.

6. [5 pts] What is the energy change of the atom when it emits light for the line H\(_f\)?
   
   A. \(1.8 \times 10^{-21}\) eV
   B. \(5.4 \times 10^{-13}\) eV
   C. \(2.9\) eV
   D. \(3.7\) eV
   E. Not enough information

   \[
   \frac{E}{h} = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \text{ Js}}{4.84 \times 10^{-9}} = 2.86 \text{ eV}
   \]

7. [4 pts] Which of the following statements is correct?
   
   A. The emission lines are generated by absorption by the atom of the photon of that wavelength.
   B. The emission lines are generated when the atom drops from an excited state to a state closer to the ground state.
   C. The emission lines match the wavelength of the electron orbiting the nucleus.
   D. Each emission line represents a wavelength that corresponds to the energy state of an atom (E\(_1\), E\(_2\), etc.).
   E. Either A or D.

8. [5 pts] Which of the following statements is correct when comparing the He\(^+\) (2 protons, one electron) and the H atom (1 proton):
   
   A. The ground state classical radius of the atom will be smaller for the He\(^+\) atom.
   B. The ground state classical velocity of the He\(^+\) will be the same as that of the H atom.
   C. Instead of quantizing angular momentum in terms of \(nh/2\pi\), it will be in terms of \(nh/\pi\).
   D. The frequency of the photons radiated while the atom is in the ground state will be higher for the He\(^+\) atom.
   E. There isn’t enough information to tell.
Part II. [21 Points] The siren of a police car emits a pure tone of 1010 Hz. The speed of sound is 343 m/s.

9. [5 Points] What frequency would you hear in your car if it was at rest and the police car was moving toward you at 15 m/s?
   
   \[ f = f \left( \frac{V + V_o}{\sqrt{V - V_s}} \right) = f \left( \frac{V}{\sqrt{V - V_s}} \right) \]
   
   \[ = 1010 \left( \frac{843 + 15}{343 - 15} \right) \]
   
   \[ = 1056.1 \text{ Hz} \]

   **A.** 15.7 Hz  
   **B.** 966 Hz  
   **C.** 968 Hz  
   **D.** 1010 Hz  
   **E.** 1060 Hz

10. [5 Points] What frequency would you hear in your car if the police car was at rest and you were moving toward the police car at 15 m/s?

   \[ f = f \left( \frac{V + V_o}{\sqrt{V - V_s}} \right) = f \left( \frac{V}{\sqrt{V - V_s}} \right) \]
   
   \[ = 1010 \left( \frac{843 + 15}{343 - 15} \right) \]
   
   \[ = 1054.1 \text{ Hz} \]

   **A.** 15.7 Hz  
   **B.** 966 Hz  
   **C.** 968 Hz  
   **D.** 1010 Hz  
   **E.** 1050 Hz

11. [5 Points] If you and the police car were moving towards each other with a relative speed of 32 m/s, what frequency would you hear in your car?

   **A.** 1500 Hz  
   **B.** 1110 Hz  
   **C.** 1240 Hz  
   **D.** 1250 Hz  
   **E.** Not Enough Information to Tell

12. [6 Points] What frequency would you hear in your car if you were moving at 9 m/s and the police car was chasing you at 38 m/s?

   **A.** 925 Hz  
   **B.** 1070 Hz  
   **C.** 1100 Hz  
   **D.** 1110 Hz  
   **E.** 1210 Hz

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Away from the cop car! (as written on the board during the exam). The correct answer is B.
Part III. [22 points] A transmission diffraction grating has a 2000 slits/cm, and is illuminated by a light with \( \lambda = 550 \text{ nm} \). A very large screen is placed 2 meters from the diffraction grating.

13. [4 pts] The sizes of the slits are halved, but the spacing between them is kept the same. The angle of the second principle maxima:
   A. Increases
   B. Decreases
   C. Remains the Same.
   D. Not enough information.

14. [4 pts] All but the two center slits of the original grating are blocked off. The angle of the second principle maxima:
   A. Increases
   B. Decreases
   C. Remains the Same.
   D. Not enough information.

15. [5 pts] What is the relative phase difference between the light that comes from the upper and lower slit when one looks at a point on the screen 6 degrees below the horizontal (see figure)?
   A. 0.95 radians
   B. 6.0 radians
   C. 7.1 radians
   D. 16 radians
   E. 600 radians

16. [5 pts] Destructive interference will occur when the path difference between the two light rays is (watch significant figures):
   A. 140 nm
   B. 180 nm
   C. 280 nm
   D. 550 nm
   E. 1100 nm

17. [4 pts] How intensity of light is related to amplitude of a light wave?
   A. The intensity is proportional to the amplitude.
   B. The intensity is proportional to twice the amplitude.
   C. The intensity is proportional to the square root of the amplitude.
   D. The intensity is proportional to the square of the amplitude.
   E. The intensity is proportional to the cube of the amplitude.
IV. [20 pts. total] An electromagnetic plane wave travels in free space. The electric field vector is given by \( \vec{E} = E_o \cos(kx - \omega t) \hat{z} \) (where \( \hat{z} \) is the unit vector along the positive-z direction).

18. [4 pts] In which direction does the wave travel?

A. in the positive-x direction
B. in the negative-x direction
C. in the positive-z direction
D. in the negative-z direction
E. none of the above

The argument of the cosine function shows that as time increases, the value of \( x \) must also increase to keep the value of \( kx - \omega t \) the same and therefore stay with the same part of the wave (e.g. to remain with a particular crest).

19. [4 pts] In which direction does the electric field vector point at \( x = 0 \) and \( t = 0 \)?

A. in the positive-x direction
B. in the negative-x direction
C. in the positive-z direction
D. in the negative-z direction
E. none of the above

At \( x=0 \) and \( t=0 \), the electric field vector is

\( \vec{E} = E_o \cos(0) \hat{z} = E_o \hat{z} \).

20. [4 pts] In which direction does the magnetic field vector point at \( x = 0 \) and \( t = 0 \)?

A. in the positive-x direction
B. in the negative-x direction
C. in the positive-z direction
D. in the negative-z direction
E. none of the above

The direction of propagation is given by \( \vec{E} \times \vec{B} \). At \( t=0 \) and \( x=0 \), \( \vec{E} \) is in the +z direction and the wave is propagating in the +x direction, so \( \vec{B} \) must therefore be in the -y direction.
A different electromagnetic plane wave is shown at right. A bulb connected to a thin conducting wire is placed in the wave with the wire parallel to the y-axis. In Case Q, the bulb is centered on the x-axis. In Case R, the bulb is shifted in the y-direction from Case Q, such that the lower end of the wire is on the x-axis. In Case S, the bulb is shifted in the z-direction from Case Q.

21. [4 pts] The bulb is initially positioned as shown in Case Q. The bulb is then moved to the position shown in Case R. Does the apparent brightness of the bulb increase, decrease, or remain the same?

A. The brightness increases.
B. The brightness decreases.
C. The brightness remains the same.
D. It is not possible to determine from the information given.

This is a plane wave propagating in the +x direction, so the values of E and B are the same everywhere with the same x value. The bulb has the same x value and is parallel to E in both cases, so the current through the bulb will be the same, and it will appear equally bright.

22. [4 pts] The bulb is initially positioned as shown in Case Q. The bulb is then moved to the position shown in Case S. Does the apparent brightness of the bulb increase, decrease, or remain the same?

A. The brightness increases.
B. The brightness decreases.
C. The brightness remains the same.
D. It is not possible to determine from the information given.

Same reasoning as previous question.
V. [25 pts.] Consider two initially uncharged electrodes X and Y, both made of a material with work function \( \mathcal{W} \), fixed inside an evacuated tube. Electrode Y is illuminated with monochromatic light of frequency \( f \). The apparatus is connected to a power supply as shown. (A positive power supply voltage corresponds to the polarity shown, and a negative power supply voltage corresponds to the reversed polarity.)

It is observed that as the potential difference across the evacuated tube is varied, the ammeter does not always read zero.

23. [6 pts.] If the power supply were set to a value slightly more negative than the stopping voltage, would any electrons be emitted from electrode Y? If so, describe the path of an electron that is ejected directly toward electrode X and illustrate your response on the diagram above. Explain.

Since there is a current at some potential difference, the frequency of the light must be high enough to eject electrons. Some of those electrons will be ejected from Y directly toward X. When the power supply is set at a value slightly more negative than the stopping voltage, the initial kinetic energy of the fastest electrons is less than that required for them to make it all the way to electrode X. So they will turn around just before reaching electrode X. This is illustrated on the diagram above.

Electrode X is now replaced by electrode Z of a different size and shape as shown.

24. [6 pts.] If the power supply were set to zero, would the new circuit have a current greater than, less than, or approximately equal to the current in the original circuit under the same conditions? Explain.

Electrons are ejected from electrode Y in all different directions. More of those will hit electrode Z in this circuit than electrode X in the previous circuit. So this circuit will have a greater current than the original circuit under the same conditions. (Note: Conventional current is opposite electron flow.)

25. [6 pts.] If the power supply were set at a very high positive voltage, would the new circuit have a current significantly greater than, significantly less than, or approximately equal to the current in the original circuit under the same conditions? Explain.

When the power supply is set at a high voltage, nearly all the electrons will be attracted to electrode X or Z. So the current will be approximately equal to that in the original circuit under the same conditions.
Electrode $Z$ is now removed from the tube, and the circuit is disconnected from electrode $Y$. Electrode $Y$ is illuminated with monochromatic light of the same frequency and intensity as in the original experiment.

26. [2 pts] Will any electrons be emitted from electrode $Y$?

Yes, just as they were in problem 24.

27. [5 pts] If the intensity of the light is increased, will the number of electrons emitted increase, decrease, or remain the same? Explain. If no electrons are emitted, state so explicitly.

If the intensity of the light is increased, more electrons will be ejected, but they will not get any more energy. Increasing the intensity corresponds to increasing the number of incident photons and thus the number of ejected electrons.
Part VI. [25 Points] Optics

28. [4 pts] A microbe (a very small life form) has a spherical shape, with diameter 10 μm. If it is viewed by eye, from a distance of 25 cm, what is its angular width? Show your work.

If the near point is at 25 cm, the angle subtended is $10 \times 10^6 / 0.25$ radians, or angle $= 0.0023$ degrees

29. [7 pts] Design a simple magnifier, using a single converging lens, so that the angular width of the microbe appears to be 0.02 degrees when its image is at infinity. On the provided optic axis, neatly draw the lens and two principal rays, and show the object location with respect to the focal points. Exaggerate the angles for clarity!

Drawing should show converging lens [1 pt] with object at focal point [2 pts], 2 principal rays [2 pts], parallel extended back to infinity dotted [1 pt], clarity [2 pt].

30. [3 pts] Show how to calculate the angular width of the image by geometry, and express the focal length in terms of given quantities. You may use the small angle approximation if appropriate.

Angular size $= \tan^{-1} \frac{h}{f} \approx \frac{h}{f}$

31. [4 pts] What is the angular magnification (sign and magnitude)? Show your work.

Angular magnification $= 0.02 \text{ degree} / 0.0023 \text{ degree}$ [2pts]. Image and object both upright, so sign is +. [1 pt]

32. [7 pts] If the space between the microbe and the first surface of the lens were to be filled with water, what adjustment to the position of the lens would have to be made relative to the microbe to keep the image at infinity? Qualitative, not numerical, answers are expected. Be sure to give your reasoning. Assume both surfaces of the lens are convex.

The addition of water, which has a refractive index of 1.33 instead of 1, means the bending of rays at the first surface is reduced and the focal length increases. Hence the magnification decreases, because the magnification of a simple magnifier is $25/f$. (The
angle subtended without the magnifier is \( h/25 \), and with the magnifier is \( h/f \), when angles are small). One can also consider the water to form a single-surface concave lens with the concave surface nested with the convex surface of the glass lens. The combined focal length of a converging and diverging lens is always longer than the converging lens' (specifically, \( 1/f_{\text{total}} = 1/f_{\text{conv}} + 1/f_{\text{div}} \), and since \( f_{\text{div}} \) is negative, the effect is to make \( f_{\text{total}} \) larger than \( f_{\text{conv}} \). All that is expected here is

- Water has a refractive index larger than 1 [2 pts]
- This makes the refraction less at the first surface [2 pts]
- This makes the focal length of the lens greater [3 pts]