Pre-Lab 8

Diffraction spectra & Planck’s constant

Objectives

- Learn the principles of a diffraction grating
- Use one to measure the wavelengths of spectral lines for hydrogen; compare the latter with expectations from the Bohr atom model
- Find the relation between light frequency and photon energy (Planck’s constant)

References

   - Diffraction gratings: Section 33-8, especially Example 33-10.
   - Hydrogen spectra: Sections 36-1, 36-2.
   - Photoelectric effect: Section 34-3, Example 34-1. (Diodes: Section 38-7.)

   - Diffraction gratings: Section 28-6, Examples 28-7.
   - Hydrogen spectra: Sections 31-2, 31-3, Example 31-1.
   - Photoelectric effect: Section 30-2.

1. Grating spectrometer

Multiple slits, each a distance \( d \) apart, produce a diffraction pattern with maxima at the same angles as two slits, only much sharper. The maxima are at

\[
d \sin \theta = m \lambda \quad \text{with} \quad m = 0, \pm 1, \pm 2, \ldots
\]

2. Bohr atom and the Rydberg formula

The “semi classical” Bohr atom model prediction for the energy levels of the hydrogen atom is consistent with accurate quantum mechanical treatment, and leads to the prediction that photons can be absorbed or emitted with only the discrete wavelengths

\[
\lambda_{n,m} = \frac{1}{R \left( \frac{1}{m} - \frac{1}{n} \right)}
\]

For \( n, m \) integers. Only wavelengths of the “Balmer” series, \( m = 2, n = 3, 4, 5 \) are in the visible band. We will measure these in this lab. The constant \( R \) is the Rydberg constant, a collection of fundamental constants with the value

\[
R = 1.097 \times 10^7 \text{ m}^{-1} = 1/(91.27 \text{ nm}).
\]

The latter value is convenient for optical wavelengths.
3. The photoelectric effect

The photoelectric effect is the creation of an electric current by light. It was discovered that light hitting one metal plate can cause electric charge—electrons—to traverse a space between that plate and another nearby. If a voltage is applied between the plates, the electrons can be repelled. At the voltage threshold where current ceases to flow, only the most energetic electrons can reach the opposite plate, and by noting this voltage, this maximum kinetic energy can be measured. This maximum kinetic energy energy is

\[
K_{\text{max}} = \frac{1}{2}mv^2 = eV_{\text{stop}},
\]

(3)

The voltage \(V_{\text{stop}}\) is the stopping voltage and \(e\) is the charge of an electron, \(1.6022 \times 10^{-19}\) C. The circuit diagram is shown below.

![Circuit Diagram](image)

The charges that are released from one plate and travel to the other are called photoelectrons. The remarkable discovery about the photoelectric effect is that \(K_{\text{max}}\) did not depend on how bright the light was, but on what the color of the light was. This observation supported two ideas:

- Each electron gets released through the absorption of the same amount of energy per electron.
- The energy of each absorption is equal to a certain amount needed to liberate the electron from the metal plus the amount needed to increase the kinetic energy of the electron, and the maximum kinetic energy is determined by the color of the impinging light.

The model of light that accounts for these ideas is that light energy is divisible down to a certain minimum amount called a photon, wherein each photon of light gets completely absorbed when exactly one electron gets released. If the color of the light is too red (or infrared), then NO electrons get emitted no matter how intense the light is, but the more towards the blue (or ultraviolet) end of the spectrum the light is, the greater the electron’s kinetic energy is. What does depend on the light intensity is the total number of electrons released, that is, the maximum current in the circuit.
Mathematically, the equation that describes the observation is

$$K_{\text{max}} = hf - W_0$$  \hspace{1cm} (4)

The maximum kinetic energy if the photoelectron is directly proportional to the frequency of the light, as long as the frequency is above a minimum value \( f_0 = W_0/h \). Since kinetic energy cannot be negative, if the light frequency (color) is too low, no electrons are emitted. The energy \( W_0 \) depends on the type of metal (i.e., gold, aluminum, tungsten, etc.), and is called the work function of the metal: it is the minimum energy needed to eject a photoelectron from the metal.

The proportionality constant \( h \) is Planck’s constant. Each photon has energy

$$E = hf$$  \hspace{1cm} (5)

In the experiment, one measures \( V_{\text{stop}} \) as a function of the light frequency \( f \), and plots the result. The plot follows the function

$$V_{\text{stop}} = \left( \frac{h}{e} \right) f - \frac{W_0}{e}$$  \hspace{1cm} (6)

This is a straight line whose slope is equal to \( h/e \). If we interpret the voltage as the energy of each electron, that is, we read the \( y \) axis in units of electron-volts, then the slope is simply Planck’s constant in units of eV-s (electron-volt seconds). Remember that a battery operating at a voltage \( V \) gives each electron, with (negative) charge \( e \), an energy of \( eV \). (That’s \( e \) times \( V \); it may be confusing that we often use “eV” units to describe the energy. An electron from a 1-Volt battery will have 1 eV of energy.) Since 1 V = 1 J/C (joule per coulomb), 1 eV = 1.6022×10^{-19} J.

### 4. Measuring Planck’s constant using colored LEDs

The photoelectric effect experiment is tricky: it requires sensitive current meters, strong, monochromatic light sources, and specially-made components (the phototube). But, similar physics can be found in a common electronic item: a colored LED (light-emitting diode). An LED is a “semiconductor device” made up of the close bonding of two dissimilar crystalline materials. An electron traversing the boundary between the two materials can have its energy change abruptly. When a small voltage is applied to the LED, very little current flows and no light is emitted; when the applied voltage reaches a threshold value, the current rises rapidly and the LED glows. A typical current vs. voltage curve is shown here. As in the photoelectric effect, the threshold voltage depends on the color of the light, except in this case, one is using a current (in the LED) to generate light, rather than using light (hitting a phototube) to generate current, so in some respects, the phenomenon is the inverse of the photoelectric effect: Energy is given to the electrons by the battery. When they flow through the LED, they give up a fixed amount of energy and emit light whose color depends on that energy. In the photoelectric effect, the light gives energy to the electrons, causing them to flow in the circuit. In both cases, the relationship between the color and the energy depends on Planck’s constant.
The circuit used in the experiment is shown below. Note its similarity with the photoelectric effect circuit.

In the experiment, one sets the power supply so that a certain current flows through the LED, and then one measures the voltage across the LED. This gives the energy lost per electron (i.e., in units of eV). By means of a spectrometer (that uses a diffraction grating) one measures the wavelength of the color emitted by the LED. From the wavelength and the speed of light, the frequency is determined. (Remember: \( f = \frac{c}{\lambda} \).) A plot of voltage versus frequency should have a slope equal to Planck’s constant.

Unfortunately, the threshold voltage is not a sharply determined in the LED experiment as it is in the real photoelectric effect, so results from this experiment are not as clean as in that experiment. The problem is that the electrons in the LED can give up energy in other ways (by heating the LED, mainly). So, the threshold for light is not a sharp corner in the current/voltage curve (note the smooth curves in the LED graph). It is true that the voltage across the LED must be at least as big as \( hf \). In the experiment, what you will do is adjust the power supply to create a current of 10 mA (0.010 A), and then measure the voltage across the LED. This measurement is indicated in the graph showing the LED current/voltage curves.