Of Ice and Statisticians†:
Interpreting Measurements of Arctic Sea Ice Thickness

Don Percival

Applied Physics Laboratory (APL)
Department of Statistics
University of Washington, Seattle

†with apologies to William Shakespeare and John Steinbeck
Considering a Career in Statistics?  
You Might Be Interest to Know That . . .

- according to a comprehensive ranking of 200 different jobs by JobsRated.com, the three best professions are
  1. mathematician
  2. actuary
  3. statistician!!!

- the three worst professions are
  198. taxi driver
  199. dairy farmer
  200. lumberjack
Why I Like Being a Statistician

• in exchange for providing help with statistics, get to work with highly motivated folks passionate about their fields of expertise
• since statistics is used in a wide range of applications, have an opportunity to learn something about many different areas (don’t have to get ‘stuck’ in any one particular area of interest)
• some problems I have had a chance to work on:
  – assessing performance of atomic clocks
  – deciphering Martian annual atmospheric pressure cycles
  – characterizing vertical shear and turbulence in the ocean
  – assessing effect of hormone therapy on menopausal transition
  – forecasting hazards to coastal communities due to tsunamis
  – interpreting thickness of Arctic sea ice (today’s topic)
Some Background

- joint effort with Drew Rothrock (APL), Tilmann Gneiting (Department of Statistics), Mark Wensnahan (APL) and Alan Thorndike (Department of Physics, University of Puget Sound)
- scientific question of interest: has the average thickness of Arctic sea ice declined significantly over the past 30 years?
- thickness can be deduced from measurements of draft (submerged portion of sea ice)
- draft measured using upward-looking sonars on submarines
- our effort differs from previous ones by use of
  - a new statistical model for draft measurements
  - newly archived data for submarine cruises from 1975 to 2001 (almost doubling the amount of available data)
Ice Draft from Upward-Looking Sonar

101 data points, each a 50-km average.

Profile data in archive. 121,000 km

(Wensnahan et al., EOS, Jan., 2007)
Submarine Cruises

- NSIDC has draft data from 121,000 km of cruise tracks
- New data (2006) in red added 81%.

Data are archived at the National Snow and Ice Data Center
[Google: NSIDC]
Paper Charts (Analog) are scanned and a digital trace extracted.

1600 rolls: 36 km

(Wensnahan & Rothrock, GRL, 2005; Wensnahan et al., EOS, 2007)
Model for Sea Ice Thickness Data

- let $\overline{H}_{x,t}$ represent average of 1 km measurements taken at location $x$ and time $t$ ($x = [0, 0] = \text{Pole}$ & $1975 \leq t \leq 2001$)
- let $\tau$ represent time relative to start of year
- assume so-called ‘multiple regression’ model:
  $$\overline{H}_{x,t} = C + I(t) + A(\tau) + S(x) + \epsilon_{x,t},$$
where
- $C$ is the overall mean ice thickness
- $I(t)$ is the interannual variation (from one year to the next)
- $A(\tau)$ is the variation within a year (annual cycle)
- $S(x)$ is the spatial field
- $\epsilon_{x,t}$ is an error term
Understanding the Statistical Properties of $\bar{H}_{x,t}$: I

- at a given time $t$ and location $x$, we form $\bar{H}_{x,t}$ by averaging together $L = 1000$ basic measurements $H_{x,t,l}$, $l = 1, 2, \ldots, L$
- each basic measurement $H_{x,t,l}$ comes from the upward-looking sonar and is the ice thickness averaged over a 1 meter patch
- assume each $H_{x,t,l}$ comes from a population with an unknown mean $\mu$ and unknown variance $\sigma^2$ (i.e., a standard deviation of $\sigma$), where $\mu = C + I(t) + A(\tau) + S(x)$
- since basic measurements $H_{x,t,l}$ and data $\bar{H}_{x,t}$ are related by
  \[
  \bar{H}_{x,t} = \frac{1}{L} \sum_{l=1}^{L} H_{x,t,l},
  \]
  $\bar{H}_{x,t}$ is a sample mean that estimates the unknown $\mu$
Understanding the Statistical Properties of $\overline{H}_{x,t}$: II

- can estimate unknown variance $\sigma^2$ using the sample variance:

$$\hat{\sigma}^2 = \frac{1}{L - 1} \sum_{l=1}^{L} (H_{x,t,l} - \overline{H}_{x,t})^2$$

- $\sigma^2$ is the variance associated with each individual $H_{x,t,l}$

- if the $H_{x,t,l}$ came from a random sample, theory says variance associated with the sample mean $\overline{H}_{x,t}$ is $\sigma^2/L$; i.e., the variability in $\overline{H}_{x,t}$ would decrease at a rate given by $L^{-1}$

- alas, standard statistical theory is problematic because $H_{x,t,l}$ cannot reasonably be regarded as coming from a random sample

- Q: what can go wrong if you don’t have a random sample?
Proper Use of Standard Statistical Theory: I

- in late March of every year, the Acme Beer Corporation (ABC) sends the Ninety-Nine Company (NNC) three packages of beer, with 33 bottles in each package, for a total of 99 bottles of beer (considered to be a random sample from a population of beers)
- NNC has 99 employees, identified by \( l = 1, 2, \ldots, 99 \)
- at 5PM on 1 April of each year, employee \#1 opens the three packages and takes out the 99 beers, pouring the contents into 99 glasses labelled by \( l = 1, 2, \ldots, 99 \)
- employee \#l takes glass \( l \), but, prior to drinking the beer, pours it through a beer analyzing machine (BAM), which measures % hop content \( H_l \) (with no error and no loss of beer)
- employee \#99 sends data \( H_1, H_2, \ldots, H_{99} \) back to ABC
Proper Use of Standard Statistical Theory: II

• based upon $H_1, H_2, \ldots, H_{99}$, an ABC statistician\(^\dagger\) computes the sample mean $\overline{H}$, the sample variance $\hat{\sigma}^2$ and then a 95% confidence interval for the true unknown % hop content:

$$[\overline{H} - 1.96\sqrt{\frac{\hat{\sigma}^2}{99}}, \overline{H} + 1.96\sqrt{\frac{\hat{\sigma}^2}{99}}]$$

• if the target % hop content falls in this interval, upper management is happy and gives a bonus to everyone at ABC

• following plot shows data $H_l$ for 2007 (circles), their sample mean (blue dashed line), the 95% confidence interval (blue solid lines) and the target % hop content (red dashed line)

• bonuses granted – hurray!

\(^\dagger\)quite happy, by the way, with his/her job!
Results for 2007

![Graph showing results for 2007]
Misuse of Standard Statistical Theory: I

• in 2008, disaster struck: while unloading the beer, NNC employee #13 dropped one package, destroying 33 bottles of beer
• to save the day, employee #1 came up with following scheme
  – pour 66 bottles of beer into glasses 1, 2, 4, 5, 7, 8, …, 94, 95, 97, 98, leaving glasses 3, 6, 9, …, 96, 99 empty to start with
  – pour one third of glasses 2 & 4 into glass 3
  – pour one third of glasses 5 & 7 into glass 6
  – pour one third of glasses 8 & 10 into glass 9
  – …
  – pour one third of glasses 95 & 97 into glass 96
  – pour one third of glasses 98 & 1 into glass 99
• now have 99 glasses of beer (each two thirds filled), so proceed as usual (NNC employees were drinking too much anyway!)
Misuse of Standard Statistical Theory: II

• employee #1’s scheme introduces *correlation* into the data: data for glass 3 is an exact average of those for glasses 2 & 4 etc.

• we no longer have a random sample of size 99, so treating data as such leads to potential problems, as illustrated by the following two plots
Incorrect Results for 2008

hop percentage

l (glass number)
Correct Results for 2008
Taking Correlated Measurements into Account: I

• assessing the variability in a sample mean $\overline{H}$ when dealing with correlated data requires an adjustment to the rule that the variance of $\overline{H}$ is given by $\sigma^2/L$

• depending on the exact nature of the correlation, appropriate adjustment can take different forms

• for sea ice data, considered two models for correlation
  — short-range correlation: measurements that are close in distance to one another are correlated, but correlation disappears rapidly with increasing distance
  — long-range correlation: similar to short-range case, but now correlation does not disappear rapidly with increasing distance

• as next 2 plots show, difference between these models is subtle
Simulated Data with Short-Range Correlation
Simulated Data with Long-Range Correlation
Taking Correlated Measurements into Account: II

- for short-range correlation (assuming $L$ is not too small), correction to $\sigma^2/L$ takes the form $\sigma^2/L'$, where typically $L'$ (the ‘effective number’ of data points) is less than $L$ (note that rate of decay is still $L^{-1}$, the same as for a random sample)

- for long-range correlation, correction to $\sigma^2/L$ takes the form to $\sigma^2/L^\alpha$, where $0 < \alpha < 1$; i.e., the variance of $\bar{H}$ decreases at a slower rate than if we have either a random sample or short-range correlation

- as following plot shows, empirical evidence suggests that sea ice data have long-range dependence
Sample (Circles) and Theoretical Variances versus $L$

- Blue: long-range
- Dotted: short-range
- Solid: standard theory
Results from Multiple Regression Model: I

• returning now to our multiple regression model

\[
\overline{H}_{x,t} = C + I(t) + A(\tau) + S(x) + \epsilon_{x,t},
\]

can use what we have learned about how the \( \overline{H}_{x,t} \) variables are correlated to specify the statistical properties of \( \epsilon_{x,t} \)

• fitting model to the data gives us estimates of its components

• following plot shows estimated interannual variation \( I(t) \), along with residuals (estimates of \( \epsilon_{x,t} \)) about fit (blue for January to June data, red for rest of year)
Results from Multiple Regression Model: II

- change from 1981 to 2000 is $-1.13$ m
- steepest decline ($-0.08$ m/yr) occurred in 1991
- no recovery by 2000
- much fuller data set strengthens previous results (Rothrock et al., 1999, and Tucker et al., 2001)
- multiple regression model explains 79% of variance in data (standard deviation is 0.98 m)
- unexplained variance has standard deviation of 0.46 m
- estimated standard deviation of measurement errors is 0.25 m
Concluding Comments

• statistical analysis is central to properly interpreting data in almost all areas of science and technology

• statisticians can make a huge impact on issues of central interest to society

• demand for well-qualified statisticians remains high – there is much work to be done, and hopefully you will decide to join in!
References


