Basic properties of the logarithm and exponential functions

- When I write "log(x)", I mean the natural logarithm (you may be used to seeing "ln(x)").
  - If I specifically want the logarithm to the base 10, I'll write log₁₀.
- If 0 < X < ∞, then -∞ < log(X) < ∞. You can't take the log of a negative number.
- If -∞ < X < ∞, then 0 < exp(X) < ∞. The exponential of any number is positive.
- log(XY) = log(X) + log(Y)
- log(X/Y) = log(X) – log(Y)
- log(Xᵇ) = b*log(X)
- log(1) = 0
- exp(X+Y) = exp(X)*exp(Y)
- exp(X-Y) = exp(X)/exp(Y)
- exp(-X) = 1/exp(X)
- exp(0) = 1
- log(exp(X)) = exp(log(X)) = X

Problems:

1. Simplify the following expressions
   - a) exp(4)/exp(2)
   - b) log(3X) - log(X)
c) \( \frac{\exp(X+Y)}{\exp(X)} \)

d) \( \frac{\exp(X + 3Y + 2Z)}{\exp(X - 2Y + 2Z)} \)

e) \( \log(3X^2Y) - \log(X) + \log(Z/3) \)

2. Suppose \( \log(p/(1-p)) = r \). Show that \( p = \frac{\exp(r)}{1 + \exp(r)} \).

3. In 2 (above) suppose \( -\infty < r < \infty \). What is the range of possible values of \( p \)?

4. Suppose \( h = a \cdot \exp(b) \). Find an expression for \( \log(h) \).

5. Suppose \( S = X^{\exp(b)} \) where \( 0 < S < 1 \). Find an expression for \( \log(-\log(S)) \).
Solutions

1.

   a) exp(2)
   b) log(3)
   c) exp(Y)
   d) exp(5Y)
   e) log(XYZ)

2. \( \log(p/(1-p)) = r \)
   
   \[ \frac{p}{1-p} = \exp(r) \]
   
   \[ \frac{1}{p} \frac{1}{1-p} = \frac{1}{1} \exp(r) \]
   
   \[ \frac{1}{p} - 1 = \frac{1}{1} \exp(r) \]
   
   \[ \frac{1}{p} = 1 + \frac{1}{\exp(r)} = \frac{1 + \exp(r)}{\exp(r)} \]
   
   \[ p = \frac{\exp(r)}{1 + \exp(r)} \]

3. \( 0 < p < 1 \)

4. \( h = a \exp(b) \)
   
   \[ \log(h) = \log(a) + b \]

5. \( S = X^{\exp(b)} \)
   
   \[ \log(-\log(S)) = \log(-\log(X^{\exp(b)})) = \log(-\exp(b) \log(X)) = \log(-\log(X)) + b \]