Physics 311
Special Relativity

Lecture 11:

Mass and energy.

Inelastic collisions.

Today's lecture plan

- System of particles.
- Three experiments:

  Elastic collisions: what goes in, comes out.

  Inelastic collisions: sticking together.

  Heat makes mass – can it be seen?
- Mass of a system of particles.
- Examples

From one particle – to a system

• So far we've been content with just one particle: "... a particle moving at speed ***u*** in the Rocket frame...", "... a particle's speed makes angle φ with the x' axis..."

• We will now boldly add one more particle to our picture, and call it a ***system of particles***, or simply ***system***. If we get real ambitious, we might add a third particle...

• High-energy physicists, who study collisions of particles, work with systems made up of tens, hundreds, even thousands of particles.

• One very important feature of such systems is that often the system is more than the sum of its parts. We'll see some quite literal examples of that.

Simple experiment 1: elastic collision

• A very basic mechanics experiment: two marbles suspended such that they barely touch. Draw one marble back and release it. The moving marble hits the resting one, stops; the one at rest starts moving.

Elastic collisions: energy-momentum is (obviously) conserved

• Before the collision: the moving marble carries all the momentum (call it ***p***) of the system, as well as all its kinetic energy K. The total energy of the system of two marbles is $E = 2m + K$, twice the mass plus the kinetic energy.

• After the collision the moving marble comes to rest; its momentum is now zero, its kinetic energy is zero as well.

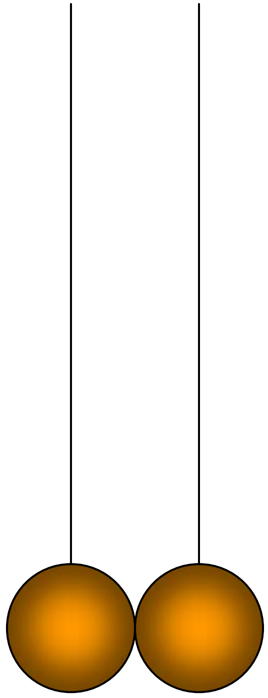
• The initially resting marble begins to move. The system's entire momentum is now carried by the second marble; so is the system's entire kinetic energy.

• Collision was elastic, i.e. by definition total kinetic energy was conserved – no heat was created, no deformation of marbles occurred.

• Total momentum was conserved, total energy was conserved, thus the energy-momentum was conserved as well!

Inelastic collision

- Well, that was too easy. The next step is an example of the inelastic collision.
- Two marbles are now replaced by two identical balls of something sticky - wax, or gum. Instead of just one, both balls are initially drawn back by equal amounts. When released, they rush towards each other, and stick...



Inelastic collisions: energy-momentum is (not so obviously) conserved

- Just before the collision: both balls carry equal and opposite momentum. Total momentum of the system is **zero**! Energy? Both balls carry equal amount of kinetic energy K, so the system's total energy is $E_{initial} = 2m + 2K$.
- After the collision the balls stick and come to rest. Total momentum is zero – as it should be.
- What about the energy? Don't we have two balls of mass m each, at rest, with total energy of $E_{final} = 2m < 2m + 2K = E_{initial}$???
- The balls are not moving, there's no kinetic energy, and thus the total energy only has the "mass" part, correct? Yes, indeed, it is correct.
- What about conservation of the energy-momentum then?!

Classical mechanics interpretation

- Classical (Newtonian) mechanics gives us a simple explanation as to where the kinetic energy disappears in inelastic collisions:
- It either goes into heat or into deformation of the bodies. Both processes convert kinetic energy of the particles' motion into some form of their internal energy.
- This indeed can be measured experimentally – the heating up, the deformation – and agrees well with the classical mechanics predictions.
- Do we need the Relativity if everything is explained by classical physics?
- Not everything! This classical explanation only works where Newtonian mechanics holds: at low velocities, low energies. When kinetic energy becomes high, there's more than just heat being generated in inelastic collisions.

Relativistic explanation: heat = mass

- Let's go back to the statement that the total (relativistic) energy of the two balls of wax at rest is equal to their mass. It is a correct statement. But *what* mass?

- We've assumed that after the balls stick together, their total rest mass is 2m, and that is what created the whole controversy. In fact, Relativity predicts that the rest mass of the two bodies has changed as the result of their sticking together! It has increased by exactly the amount of the missing kinetic energy.

- I.e. the total mass after the collision is not 2m but 2m + 2K, as if each ball of wax now weighs m' = m + K.

- So, Special Relativity tells us that heat has mass!

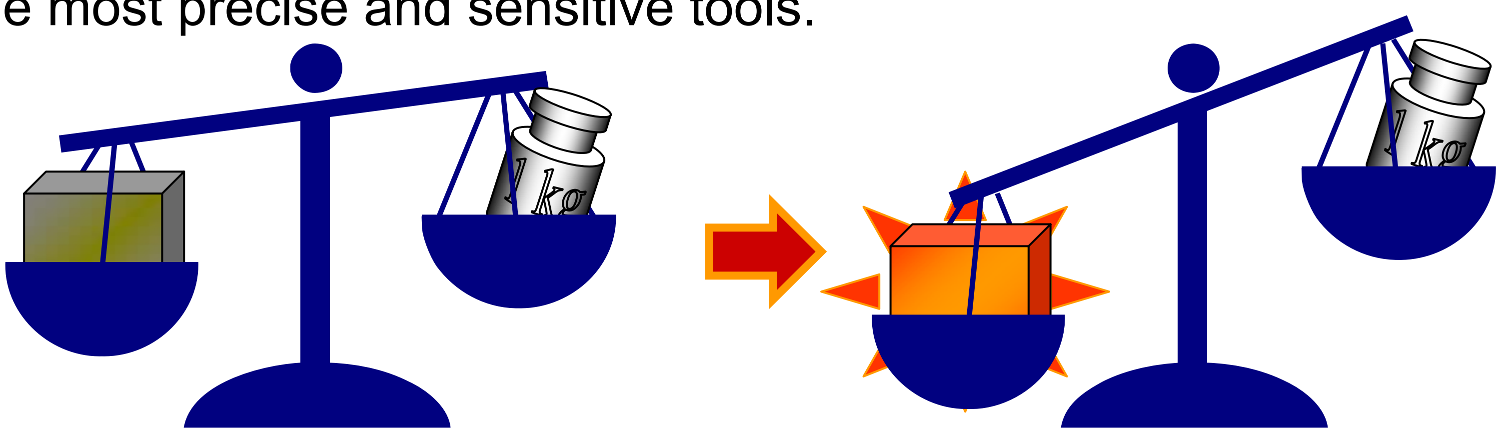
Heat has mass! (But it's hard to measure)

• If objects get heavier as they heat up, maybe we can measure that? Just put an object on a scale while heating it!

• Unfortunately, the effect is really small. To see how small it is, we now heed to remember that $E = mc^2$, and c = 299792458 m/s.

• Take a 1 kg piece of iron at t = 0 C, and at rest. It weighs 1 kg, (can anybody derive that?). Its total energy is $E = mc^2$ = 89,875,517,873,681,764 ≈ 10^{17} J. That is a lot of Joules!

• Now heat it up to 1000°C, for which approximately 440,000 J of heat will be added to the iron piece. Quite a few Joules, too, but sadly only about 5×10^{-12} of the mc^2 – an effect that we have no hope of seeing yet, even with the most precise and sensitive tools.

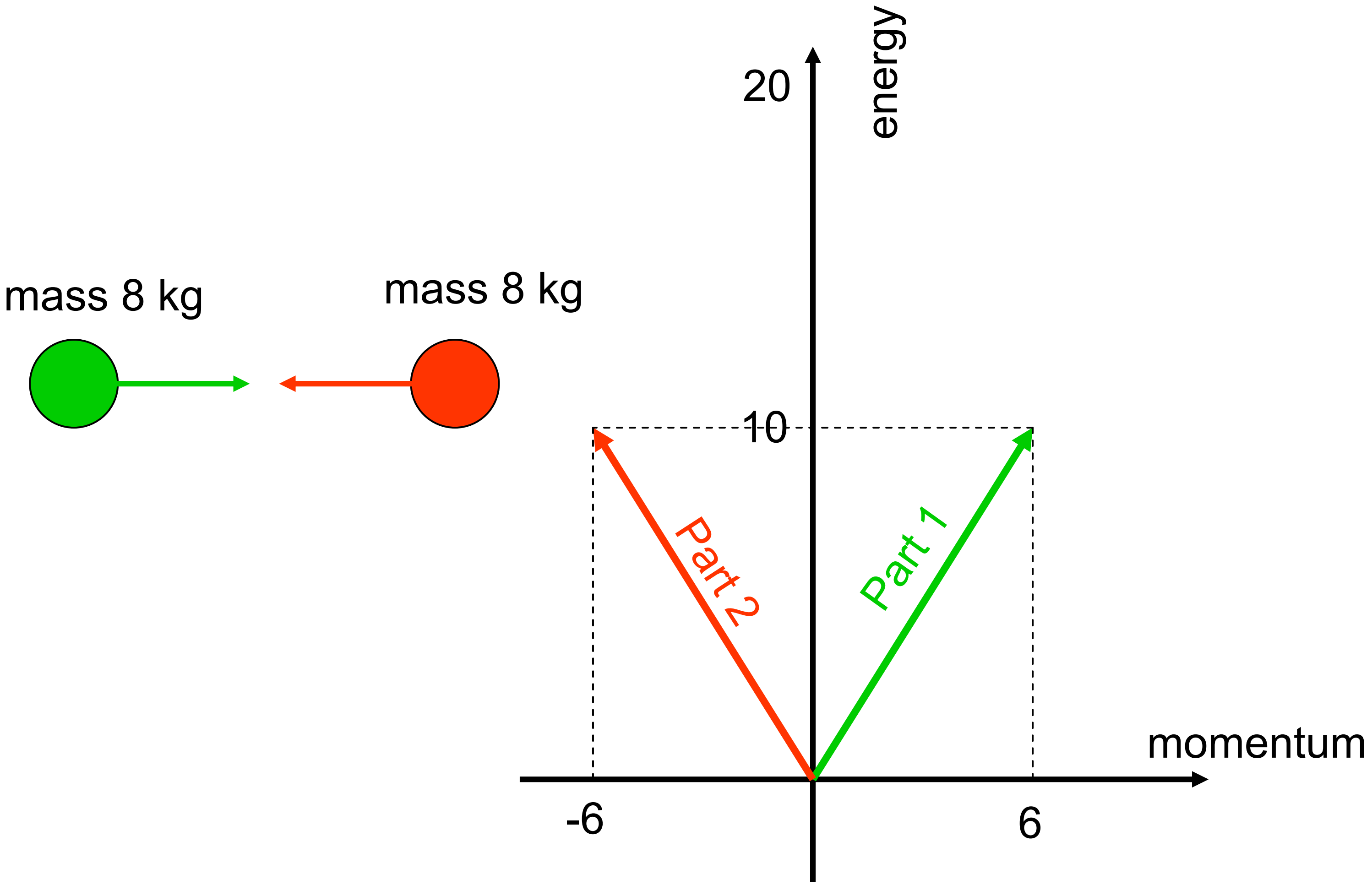


Relativistic case – momentum is of order of the rest mass.

- Not take two balls of wax an make them move really fast. So fast that their kinetic energy is 2 kg apiece.
- Is 2 kg a lot? 440,000 J of energy was ridiculously small just a second ago, so is 2 kg really anything?
- We are now back to our theory-inspired world where $c = 1$. This means that a piece of wax weighing 8 kg (quite a nice piece of wax!) would have its rest energy equal to 8 kg (or $\sim 10^{18}$ J). So 2 kg of kinetic energy is not so bad, a good fraction of the rest energy.
- Total energy of each ball is $E = m + K = 10$ kg.
- Total energy of the system? $E_{tot} = 2E = 20$ kg.
- Momentum of each ball is $p = (E^2 - m^2)^{1/2} = (100 - 64)^{1/2} = 36^{1/2} = 6$ kg. Here, I've used the energy-momentum magnitude $m^2 = E^2 - p^2$.
- Total momentum of the system? $p_{tot} = p_1 - p_2 = 6\text{ kg} - 6\text{ kg} = 0$ kg.

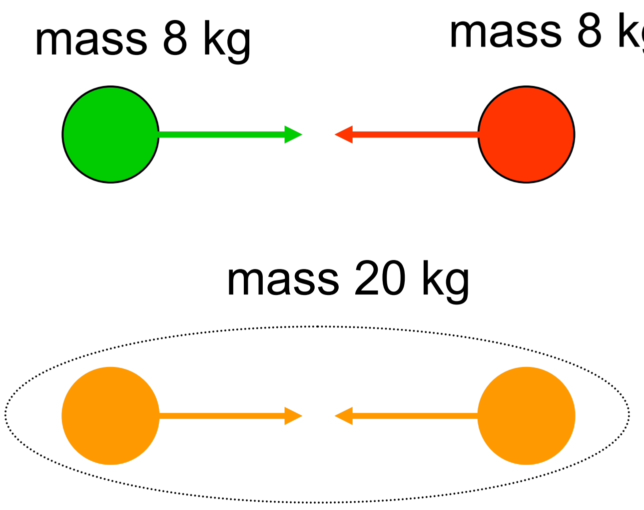
Energy-momentum diagram

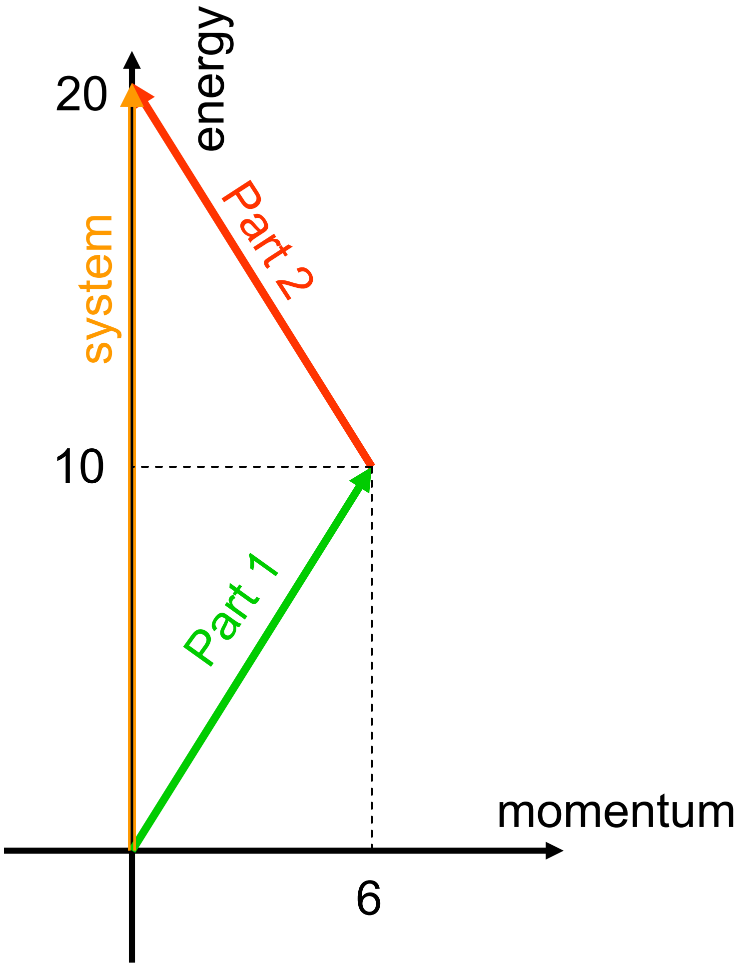
• We can draw the energy-momentum arrows of both particles on the energy-momentum map.



Rest frame of the system

• Consider the system as a whole. In the Lab frame, it is at rest – it has no net motion, even tough its two parts are moving. For energy-momentum of the system we then have $m^2 = E^2 – 0$, i.e. $m = E$.





Where is the extra mass?

• Total mass of our system is 20 kg > 8 kg + 8 kg = 16 kg. This is the case both *after* and, more surprisingly, ***before*** the collision.

• It has been mentioned that a hot object weighs more than the cold one. So we can see how two balls of wax collide to make one really hot and heavy ball of wax. But *before* they collide?.. What if the two balls miss each other? (The two momenta may be opposite, but not along the same line, the problem still holds.)

• This brings up the question: where does this extra mass come from? Where does it reside?

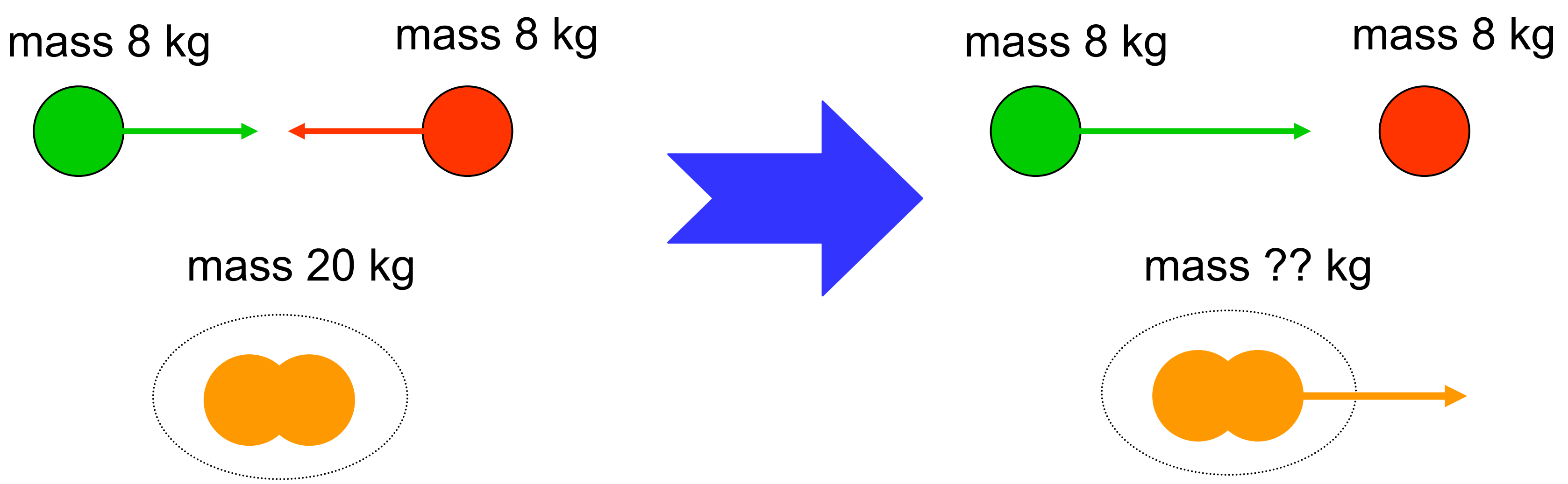
• This extra mass, quite simply, comes from the extra energy that the system possesses due to the motion of its parts. Two balls rushing towards each other is no different from atoms inside the hot ball of wax moving faster!

• However, it would be incorrect to say that each ball as a separate object carries some extra mass. The extra mass is the property of the system as a whole, and not of its parts.

System in a moving frame

• Is our system really heavier, or are we fooling ourselves? Let's see what things look like in a different frame.

• Consider a frame moving with the "red" mass. Then the "green" mass is moving towards the right, and the whole system is also moving to the right.



Moving frame: energy and momentum

• First, we need to know the velocity of each ball in the Lab frame. Recall that relativistic kinetic energy is $K = m(\gamma - 1)$, so substituting $K = 2$ kg and $m = 8$ kg we find $\gamma = 1.25$ – this number should look familiar! The velocity is easily found to be $v = 0.6$.

• Now, when we jump into the "red" frame, velocity addition formula can be used to find the "green" ball speed:

$$v_{green} = (v + v)/(1 + v^2) = (0.6 + 0.6)/(1 + 0.36) = 120/136 = 15/17$$

• For this velocity, $\gamma = (1 - (15/17)^2)^{1/2} = 2.125$, and the kinetic energy of a body of mass 8 kg is $K = 8(\gamma - 1) = 9$ kg.

• Total energy of the "green" mass: $E = 8\text{ kg} + 9\text{ kg} = 17\text{ kg}$, the "green" momentum is $p = (E^2 - m^2)^{1/2} = (289 - 64)^{1/2} = 225^{1/2} = 15\text{ kg}$.

• Total energy of the "red" mass is its rest mass 8 kg, and the "red" momentum is zero.

Moving frame: energy and momentum

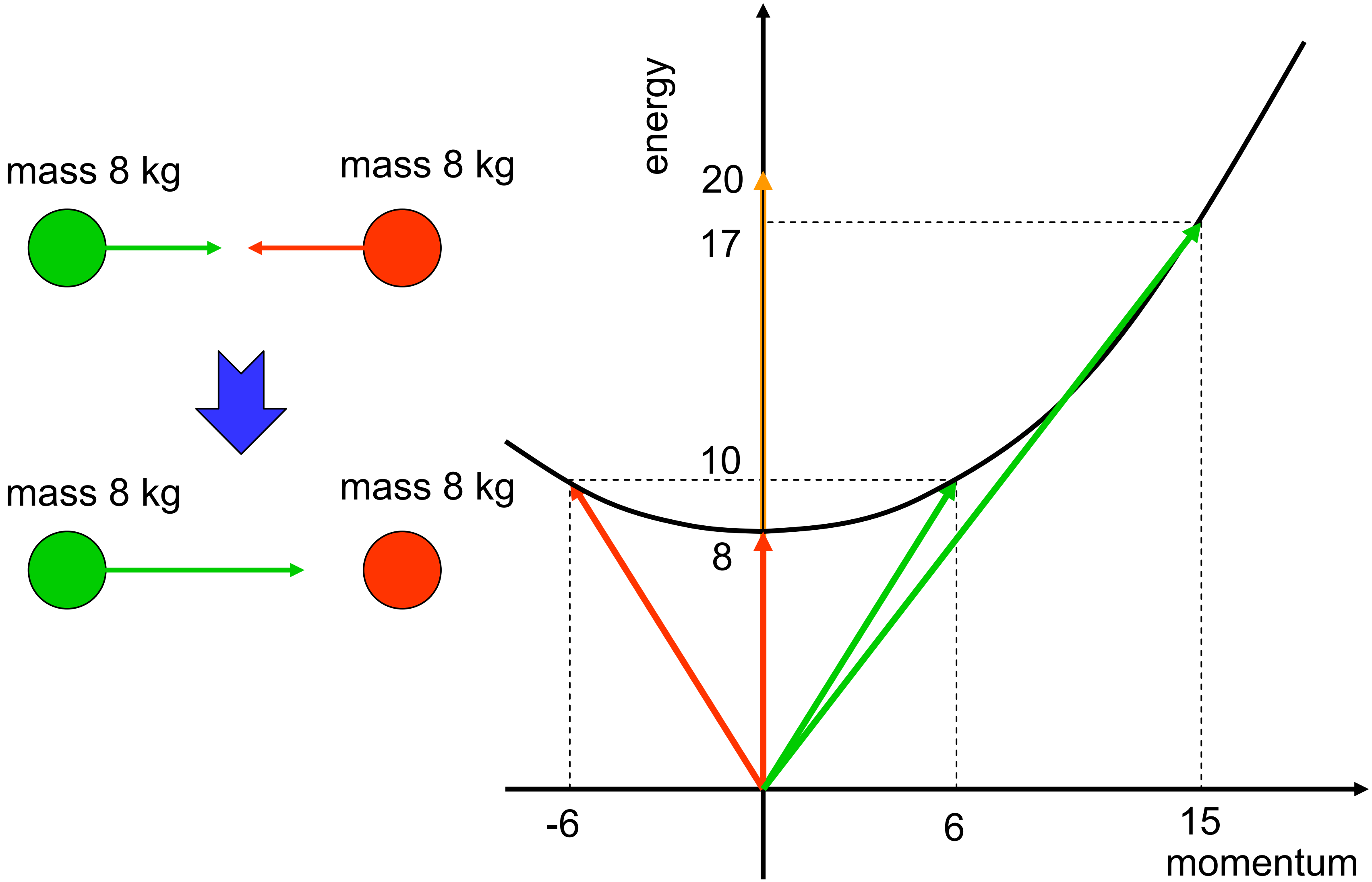
- Now for the system as a whole:
- Total energy $E_{total} = E_{green} + E_{red}$ = 17 kg + 8 kg = 25 kg.
- Total momentum $p_{total} = p_{green} + p_{red}$ = 15 kg + 0 kg = 15 kg.
- Given total energy and total momentum, we can calculate the magnitude of energy-momentum 4-vector of the system, a.k.a. it mass:

$$M = (E^2_{total} - p^2_{total})^{1/2} = (625 - 125)^{1/2} = (400)^{1/2} = 20 \text{ kg}$$

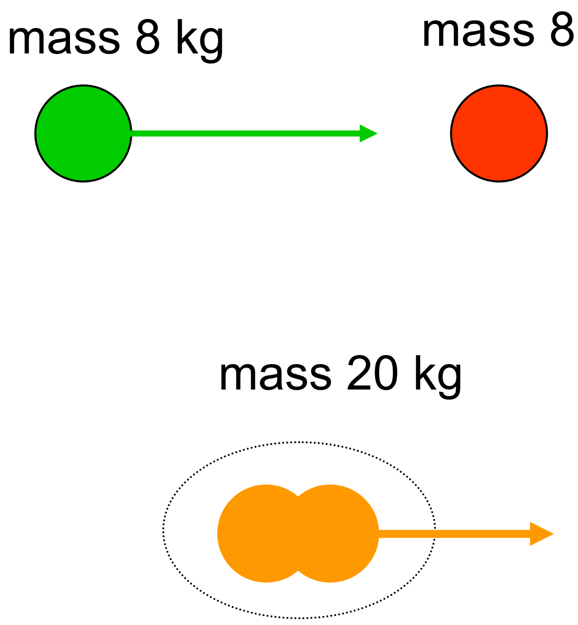
- The system mass is 20 kg - the same as before!
- Even though one may say that the choice of our moving frame was not completely arbitrary, it can be shown that for any values of m and K and in any inertial frame moving with any (allowed) velocity, the total mass of the system of particles is unchanged.

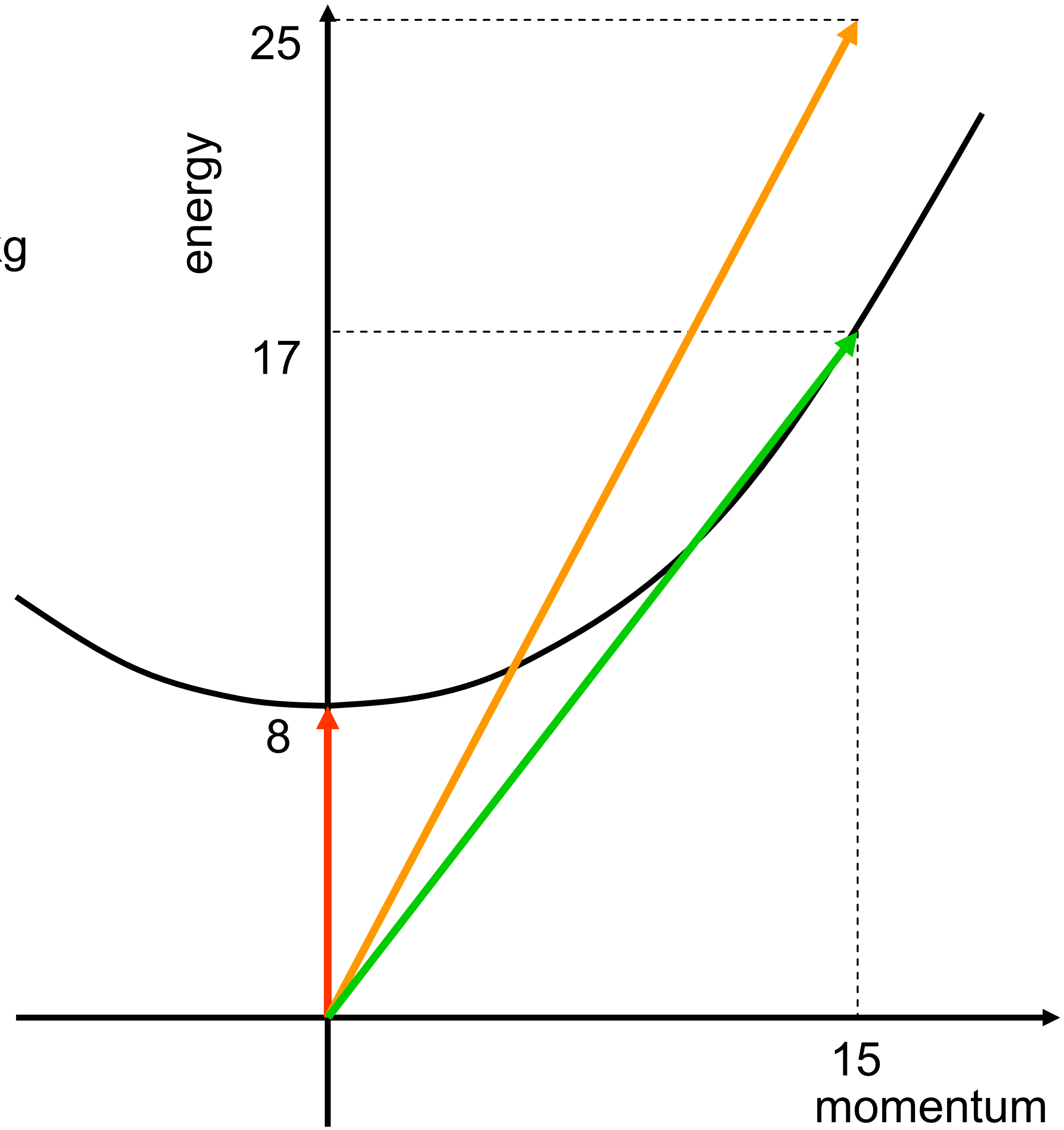
Energy-momentum diagram

- Energy-momentum diagram offers a nice way to see this.



Energy-momentum diagram: the system





Neutron capture

- Neutron capture is an important part of nuclear chain reaction. It is a classic example of inelastic collision – two incoming bodies stick together.
- Kinetic energy of the neutron as it smashes into the nucleus is significant – a few percent of neutron's rest mass. This kinetic energy comes from the neutron falling down the deep potential well of the strong nuclear force.