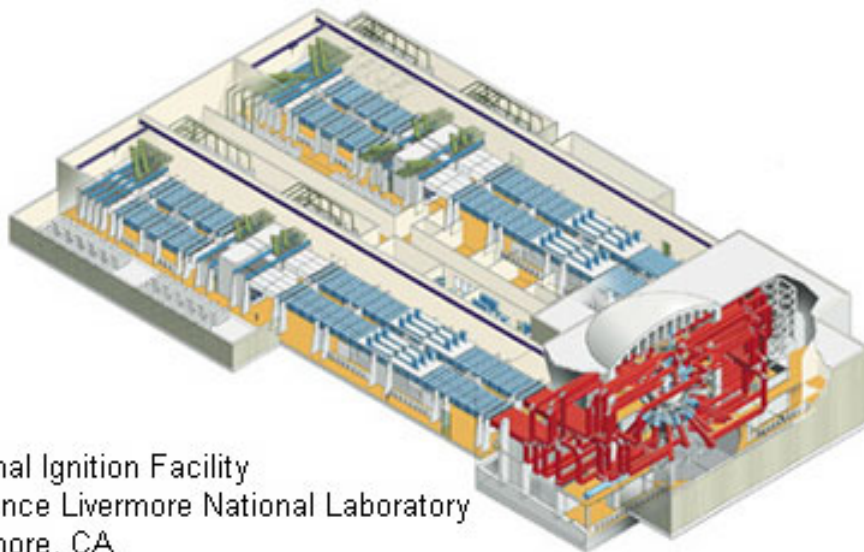# Homework #5

- 8-11 (20 points)

- 8-25 (20 points)

- 8-32 (30 points)

from "Spacetime physics"
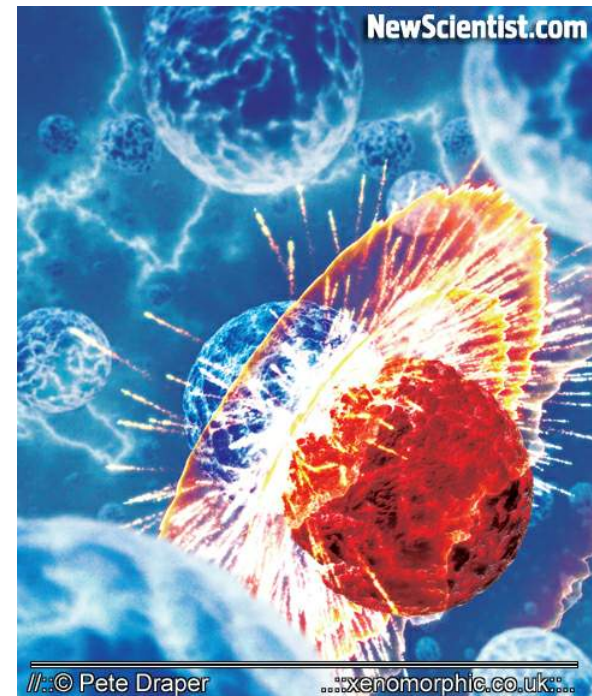
Physics 311
Special Relativity

Lecture 13:

Fusion and Fission. Annihilation.

National Ignition Facility
Lawrence Livermore National Laboratory
Livermore, CA

Today's lecture plan

- Converting mass to energy:

    - fusion

    - fission

    - annihilation

- Summary: use and abuse of the concept of mass.

Energy makes mass – and mass makes energy

• The example of photons (which we can call "pure energy") colliding to make massive particles shows one direction in the energy-mass conversion.

• Reactions going in the opposite direction exist as well – fusion, fission, annihilation.

• Direct conversion of mass to energy is very efficient because the "energy content" of mass is so high.

Electron - Positron Annihilation   NASA - Goddard Space Flight Center
Scientific Visualization Studio
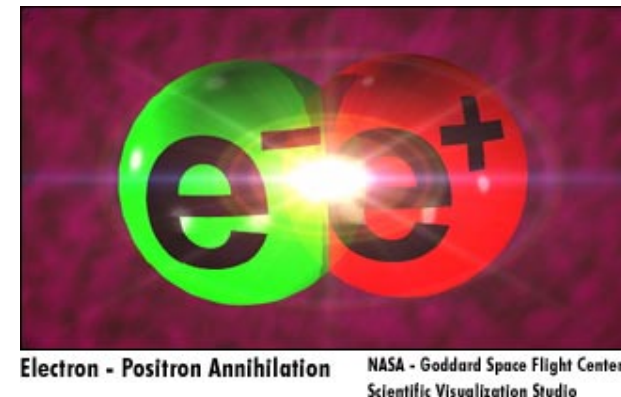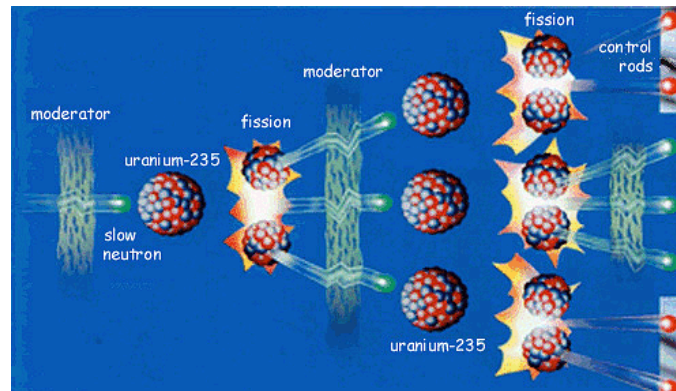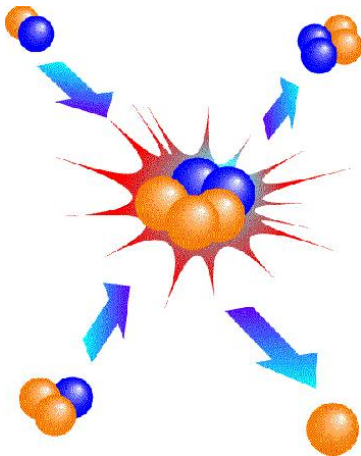
# Basis of fusion and fission: binding energy of the nuclei

• Fusion and fission are *nuclear* processes. In fusion, smaller nuclei merge (fuse) to create a larger nucleus, in fission a large nucleus splits (fissions) into smaller pieces.

• To be a source of energy, products of a nuclear reaction must have *higher* binding energy that the source nuclei.
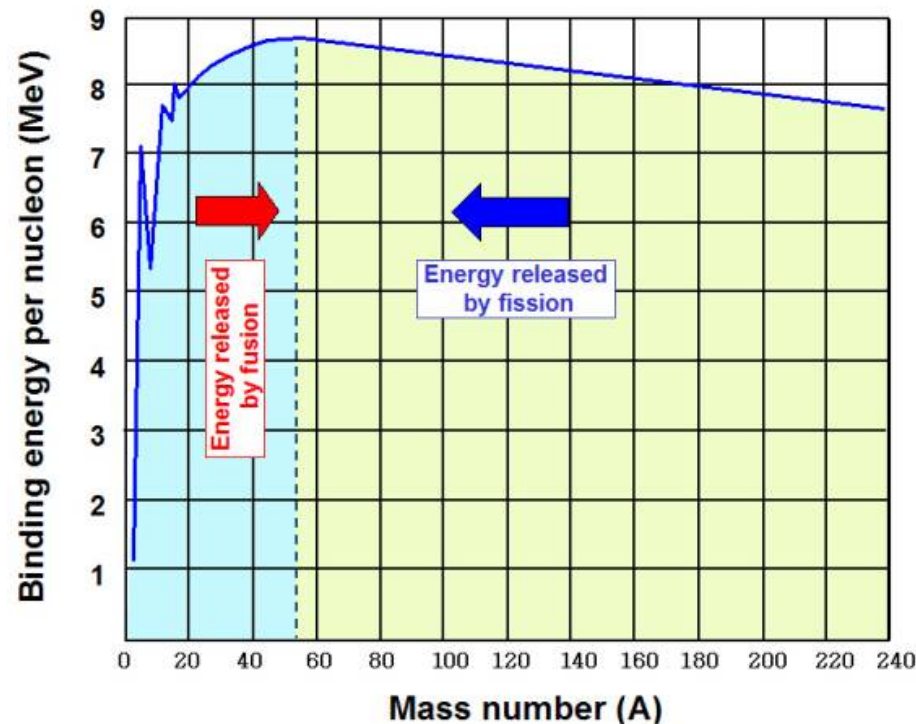
# Why going up the slope releases energy?

• Consider *nucleons* – protons and neutrons – inside a nucleus. They are held together by the *nuclear force*, also known as the *strong force*.

• Tight binding in the nucleus means the nucleons are closer together, deeper in the binding potential well. Looser binding - nucleons are less deep.

V(r)

Coulomb repulsion (for protons only)

r

nuclear
attraction
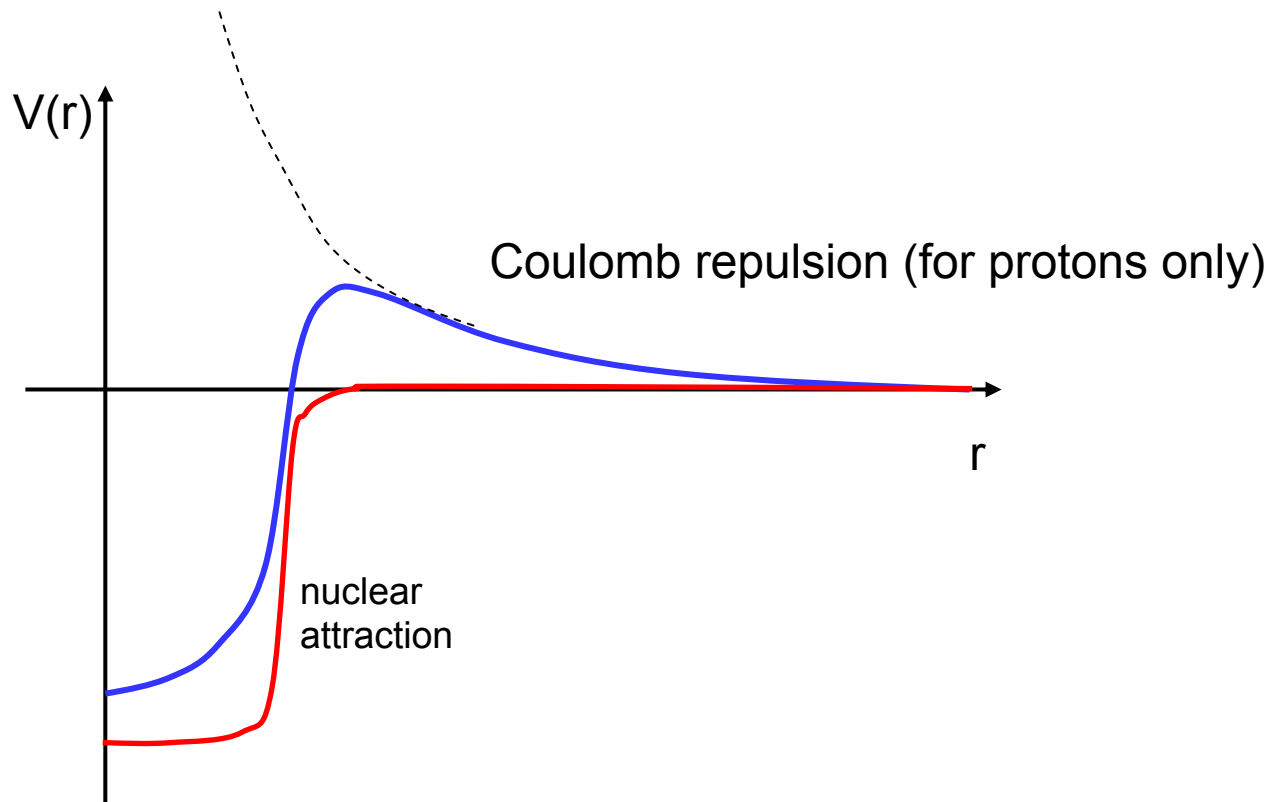
Why going up the slope releases energy?

• When nucleons go to a tighter-bound nucleus, they lose some of their potential energy; this energy first becomes the kinetic energy of the nucleons and is later converted into photons.
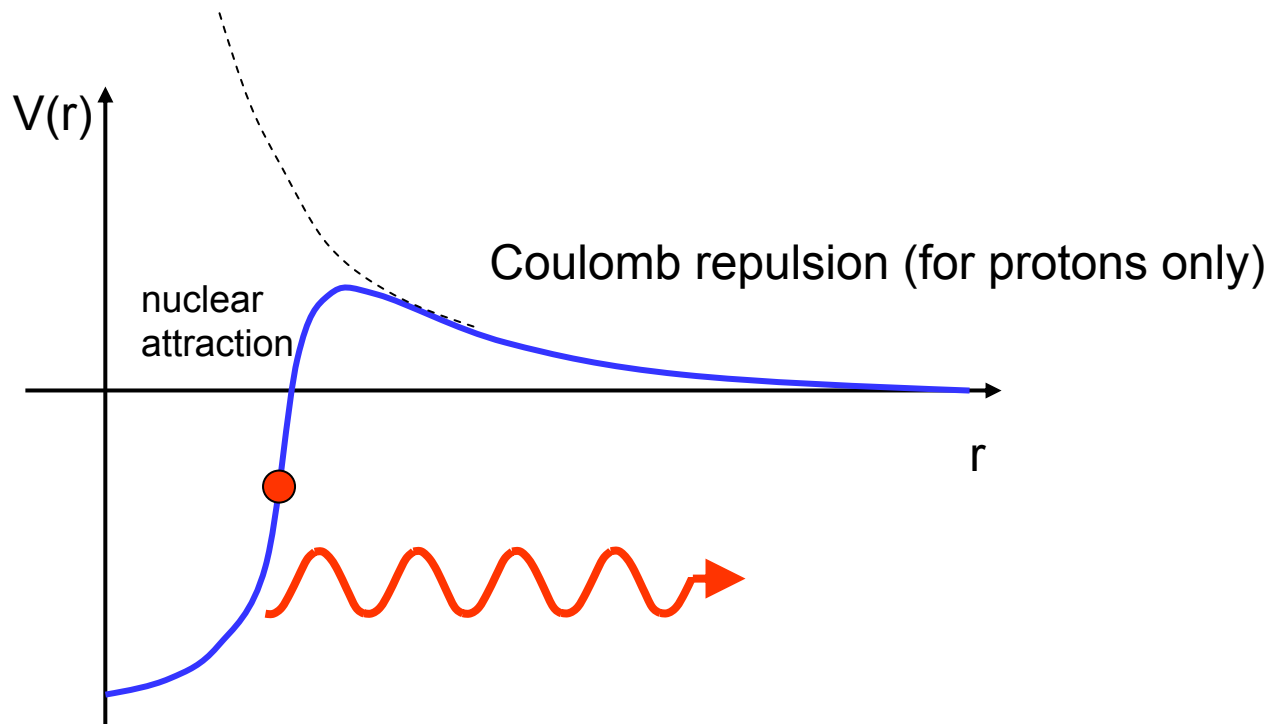
Fission

• Very large, heavy nuclei (like ^{236}U) are loosely bound. They have too many neutrons and nuclear physics tells us that having too many neutrons is not healthy in general. Particular arrangements of neutrons and protons are particularly bad.

• Fission, at least in its useable form, starts with something fairly stable, say ^{235}U. When this nucleus absorbs a neutron, it becomes a ^{236}U nucleus, which almost immediately decays:

$$^{235}U + {}^{1}n \rightarrow {}^{236}U \rightarrow {}^{87}Rb + {}^{133}Cs + 16{}^{1}n$$
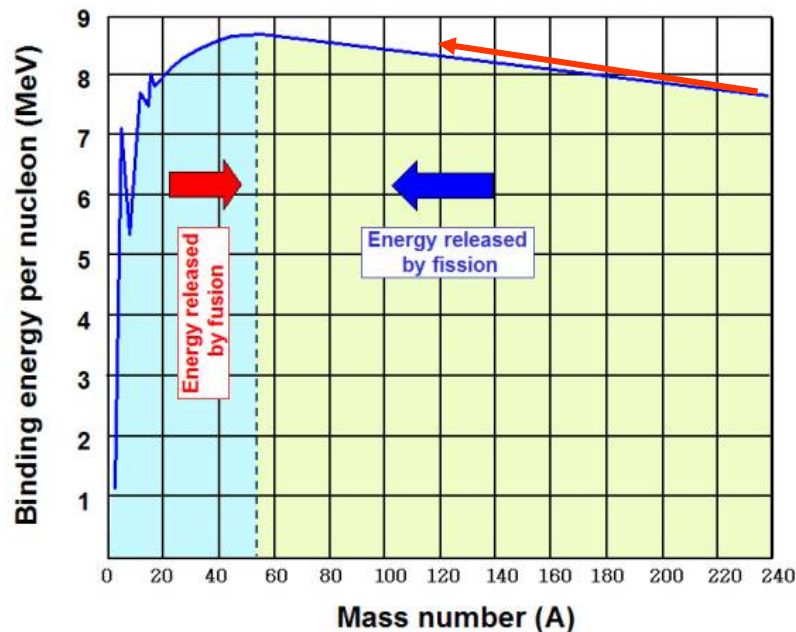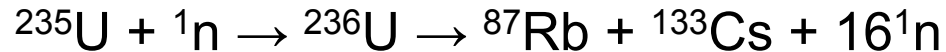
Nuclear chain reaction

• For fission to occur, a fresh supply of active nuclei (like ^{236}U) is needed. Nuclear chain reaction provides such supply from more stable nuclei (like ^{235}U) given there is enough of them.

• Critical mass: minimum mass of nuclear fuel needed for chain reaction. Fission creates neutrons which convert ^{235}U into ^{236}U which undergoes fission which creates neutrons which convert ^{235}U into ^{236}U which undergoes fission which creates neutrons which....
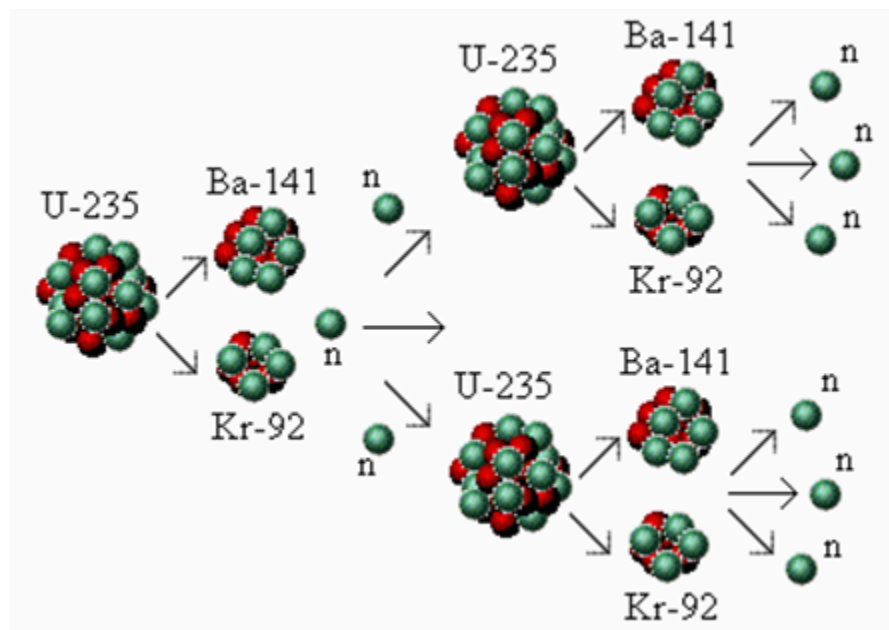
Oklo – a natural nuclear reactor

Oklo: Location

About 1.7 billion years ago a high concentration of ^{235}U in the uranium ore layer at a place now know as Oklo in Gabon created natural nuclear reactor which ran for about 100,000 years...
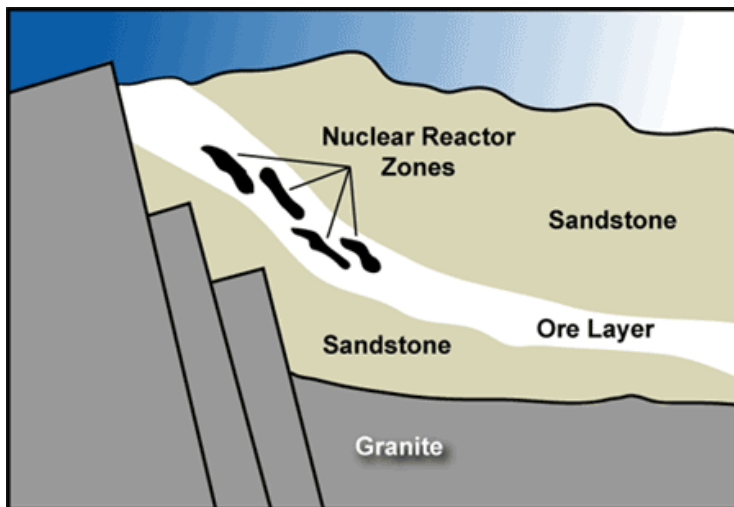
Fusion

• In a nuclear fusion reaction, two lesser nuclei combine to make a heavier one. The nuclei again go from loose binding to tight. The slope is much higher – so the energy released is much greater!
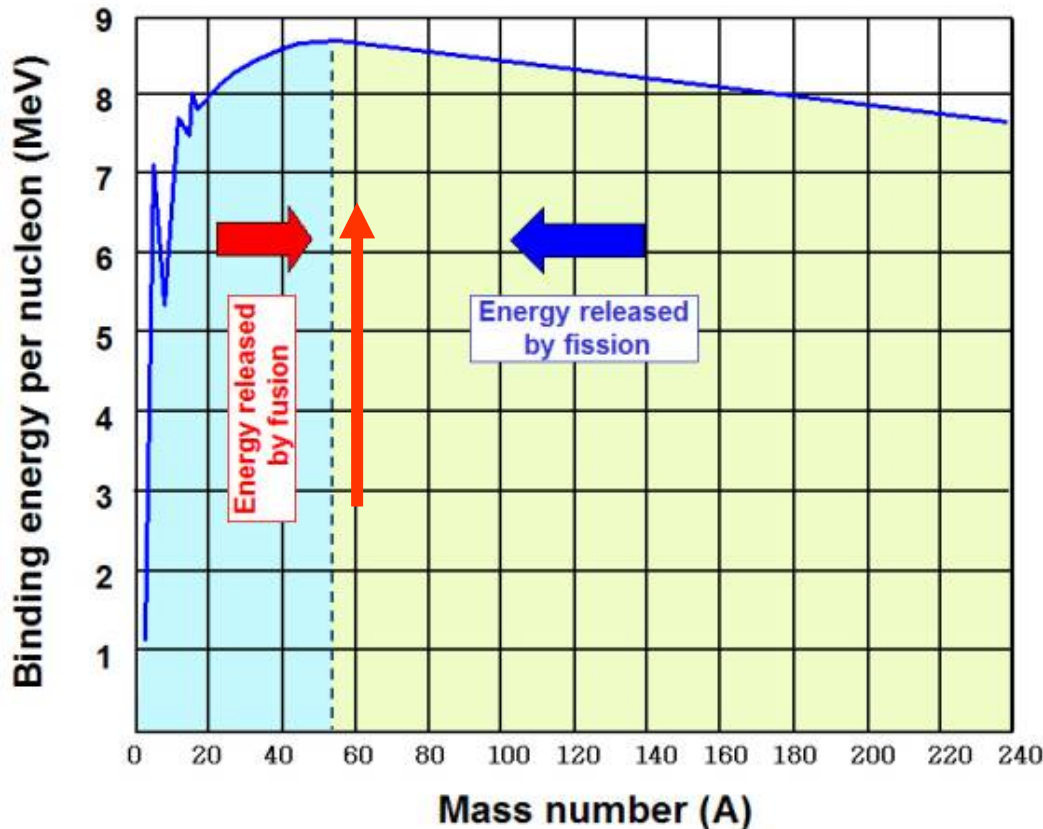
Note the scale: 0 to 9 MeV. Mass of a nucleon is about 932 MeV, so even in a fusion reaction only a fraction of one percent of the total mass is converted to energy

# Fusion: endless supply of energy. Just not quite available yet.

• Fusion energy comes from very mundane everyday stuff. Like water. Typical fusion reaction:

$$^2D + ^2D \rightarrow ^4He$$

(here ^2D is deuterium, heavy isotope of hydrogen. It is found naturally; about 1% of water is "heavy".)

• However, *controlled* fusion is still an elusive goal. The uncontrolled version of fusion exists in the form of hydrogen bomb.
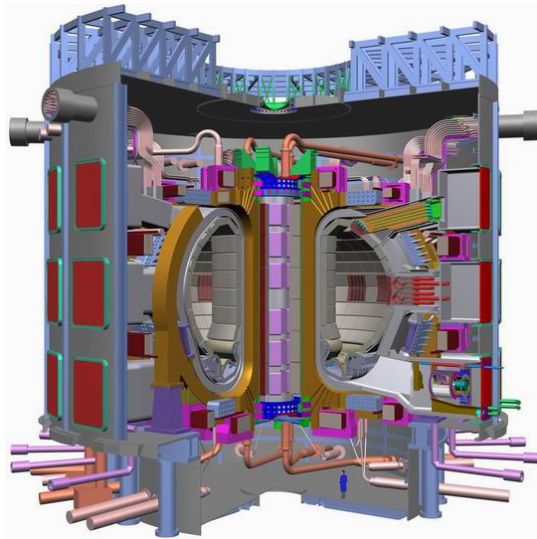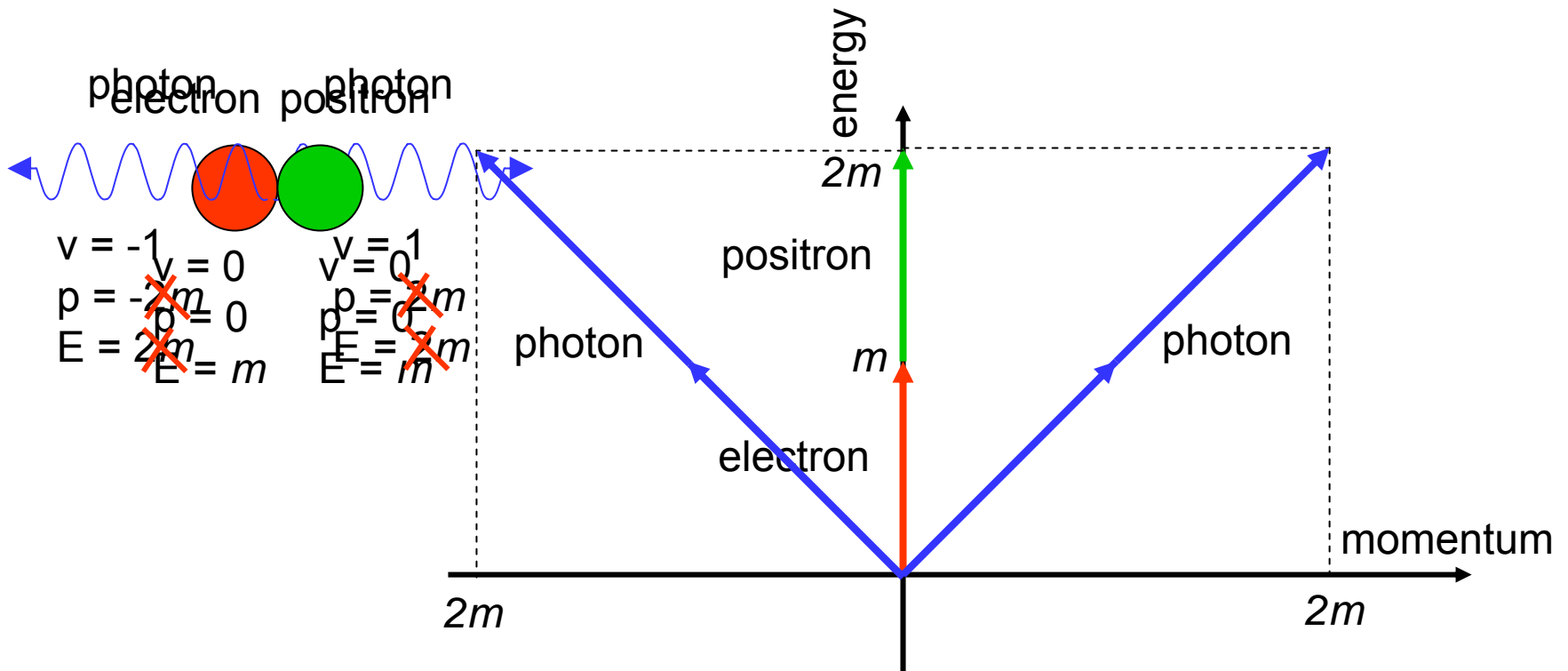
Annihilation: all mass goes into energy

• Annihilation is an interaction between a particle and its own antiparticle. Result of annihilation is electromagnetic radiation – photons.

• Simple example: an electron and a positron, both at rest (nearly), annihilate. How many photons come out?

• One big photon? Or many small ones? Conservation of energy-momentum helps us answer this question...

Energy-momentum diagram for annihilation

- Before annihilation: two particles of mass *m* each, at rest, so:
  $p = 0$, E = $2m$

- After: photon(s) of mass 0, equal energy and momentum.

Annihilation: all mass goes into energy - 2

• Annihilation is a very simple, but by no means the only possible outcome of a particle-antiparticle collision. More exciting, and more interesting from the point of view of physics, are collisions that produce new *massive* particles.

• Annihilation may be a source of energy (even more efficient than fusion – 100% of mass goes into energy), but we first need a source of antimatter.

Mass production of antimatter

• Antihydrogen can be made by combining antiprotons (the nucleus) and positrons (the "electron"). Other anti-elements are much harder to make: first, the anti-nucleus must be formed our of antiprotons and antineutrons, a very difficult process.

• Two experiments at CERN are trying to make antihydrogen in large quantities (thousands to millions of anti-atoms).

the Athena experiment

the Atrap experiment
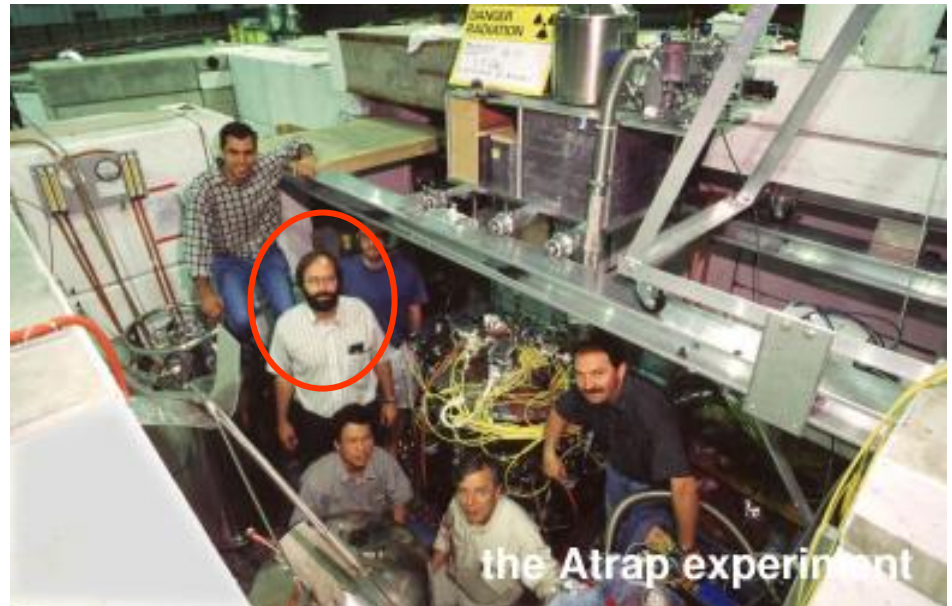
Uses of antimatter

• Antihydrogen is of interest to physicists who look for violation of certain symmetries. Hydrogen atom has been measured to an unbelievable precision; if we can now do spectroscopy on antihydrogen to a precision that is anywhere near that, small deviations can be discovered that lead to new understanding of laws of Nature. To make such high-precision measurements, many antihydrogen atoms are needed, millions or more.

• Curious fact: when NASA learned about antihydrogen production experiments, they immediately asked if CERN could send them 1 mg of antihydrogen in a bottle. Well, magnetic bottle. Still how many atoms is that? 1 gram of hydrogen (or antihydrogen) is about 6×10^{23} atoms, so 1 milligram is about 6×10^{20} atoms – a far cry from what is possible.

Mass, energy and momentum – Q&A

Q: Does the mass of an isolated system remain constant? Is it the same in all inertial frames?

A: Yes, and Yes. Mass of a system is the *magnitude* of its energy-momentum 4-vector. Because of energy-momentum conservation, this quantity remains the same no matter what happens within the system. A magnitude of a 4-vector is a property that is also invariant under Lorentz transformations, thus it is the same in all inertial frames.
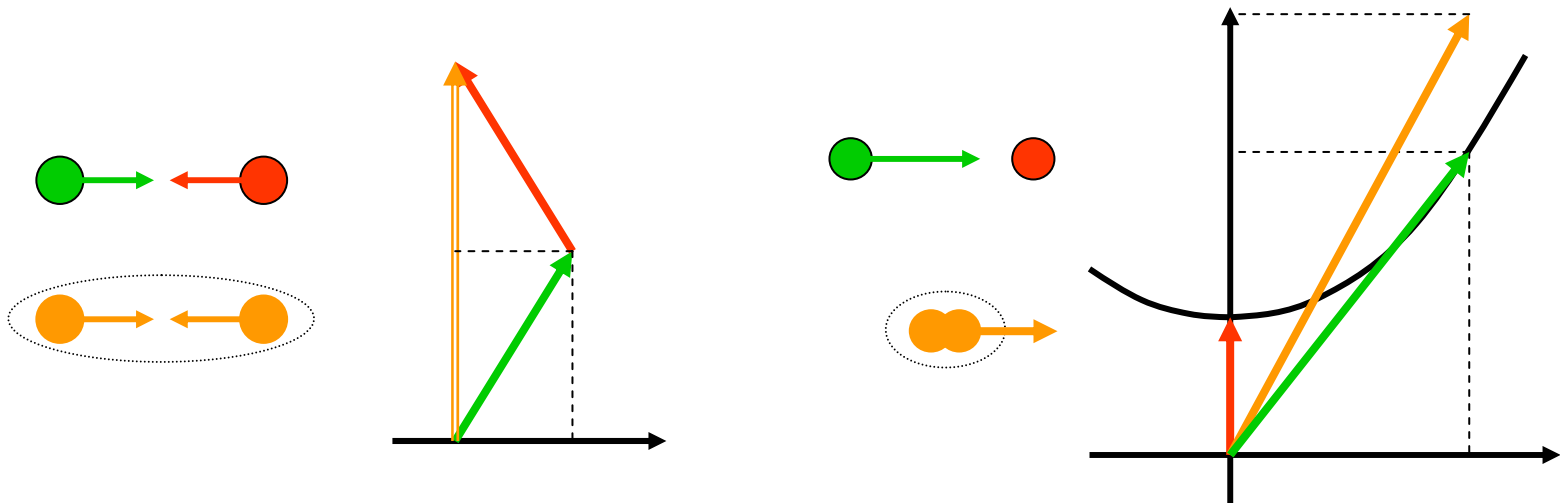
Mass, energy and momentum – Q&A

Q: Are energy and momentum of an isolated system the same in all frames?

A: No. Energy of the system is $E = m + K = m\gamma$, while momentum is $\boldsymbol{p} = m\boldsymbol{v}\gamma$. The stretch factor γ is different in different inertial frames.
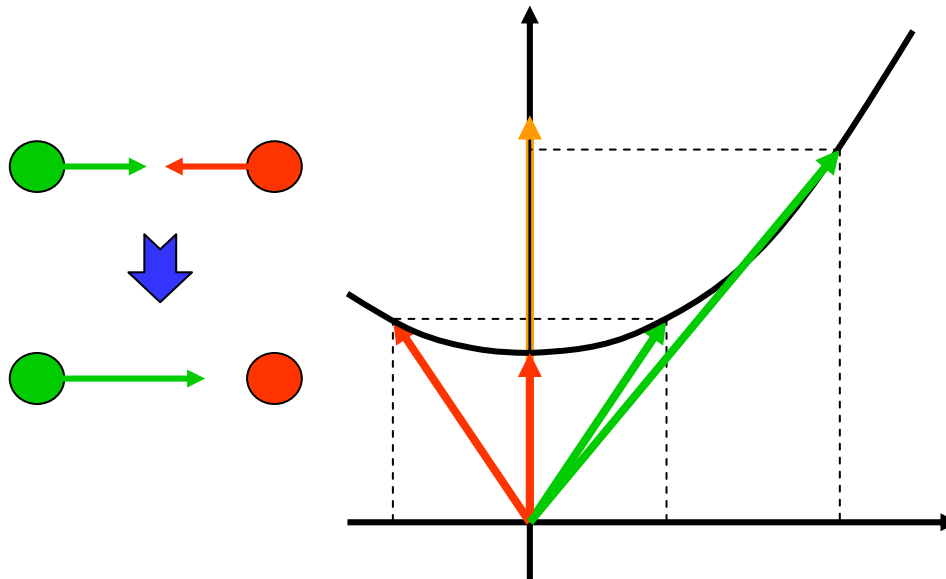
Mass, energy and momentum – Q&A

Q: Do massless particles (photons) have energy? Can massless particles add mass to a system?

A: Yes, and Yes. Energy of a massless particle is all in its motion, so we can say that it is equal to its kinetic energy K, and is also equal to the magnitude of its momentum $|\boldsymbol{p}|$. When considered a part of a system, a photon adds mass to it, proportional to the photon's energy (momentum).
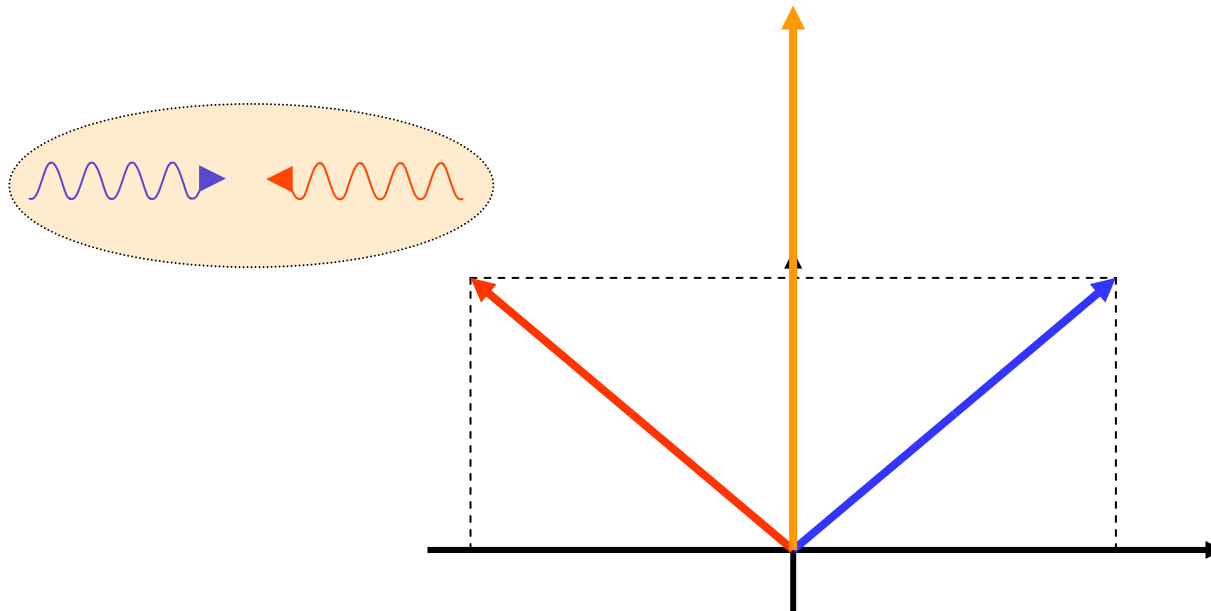
Mass, energy and momentum – Q&A

Q: If mass of a system is conserved, then is it true that individual masses of its constituents, and the constituents themselves, also remain the same?

A: Not necessarily. In inelastic collisions, objects heat up and gain mass. In annihilation, massive objects convert into massless photons. In pair production, massless particles create massive ones.
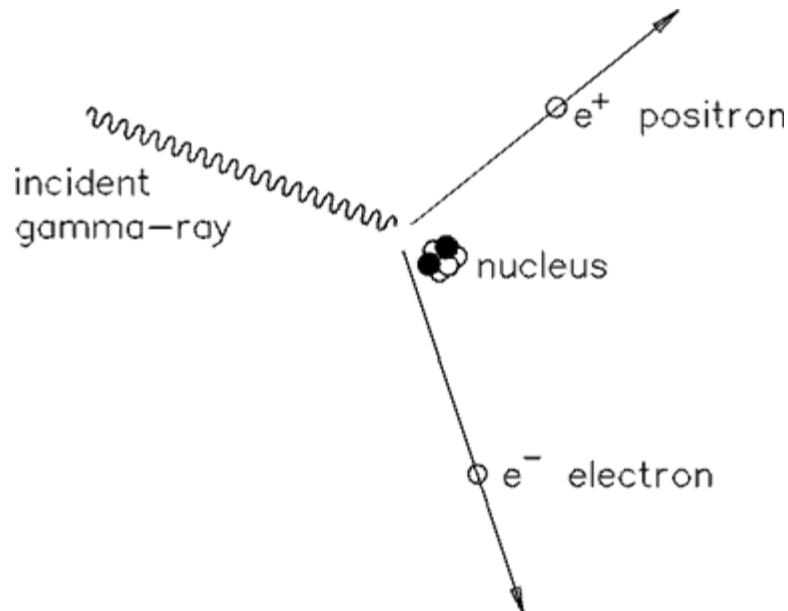
Mass, energy and momentum – Q&A

Q: What is the meaning of the statement that mass and energy are equivalent?

A: At zero momentum, energy of an object is the same as its mass (the *rest mass*), when proper units are used. If the object is moving, its total energy is increased by the amount of the kinetic energy of motion; the object's rest mass remains the same in all inertial frames.

Q: What about "relativistic mass" – doesn't an object moving at near speed of light become heavier?

A: No! Mass of an object is the magnitude of its energy-momentum, and is the same in any and all inertial frames. Do you think that if you move at $v = 0.99$ you will feel heavier? Let me assure you, even as we speak we are moving vary-very fast with respect to some high-energy cosmic ray particles, or some far-away galaxies.

Mass, energy and momentum – Q&A

Q: How do we measure mass of a system?

A: Weighing is a good way to measure mass. Gravitational pull of Earth is proportional to the mass, so... Make sure you're weighing the entire system, not just its parts. If the system is too big (or too small) to fit on a scale, use other methods – gravitational red shift, for example.

To Earth