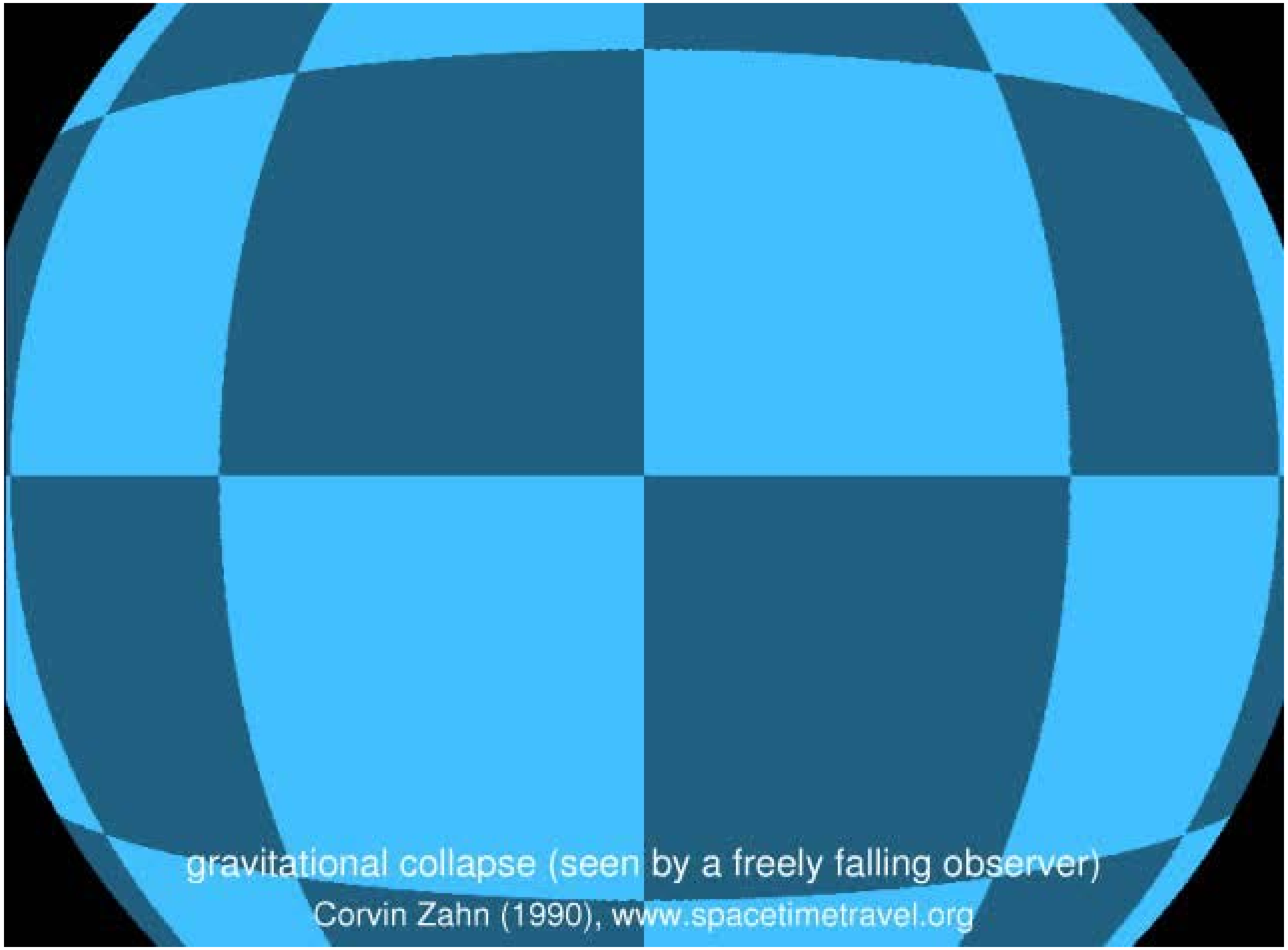
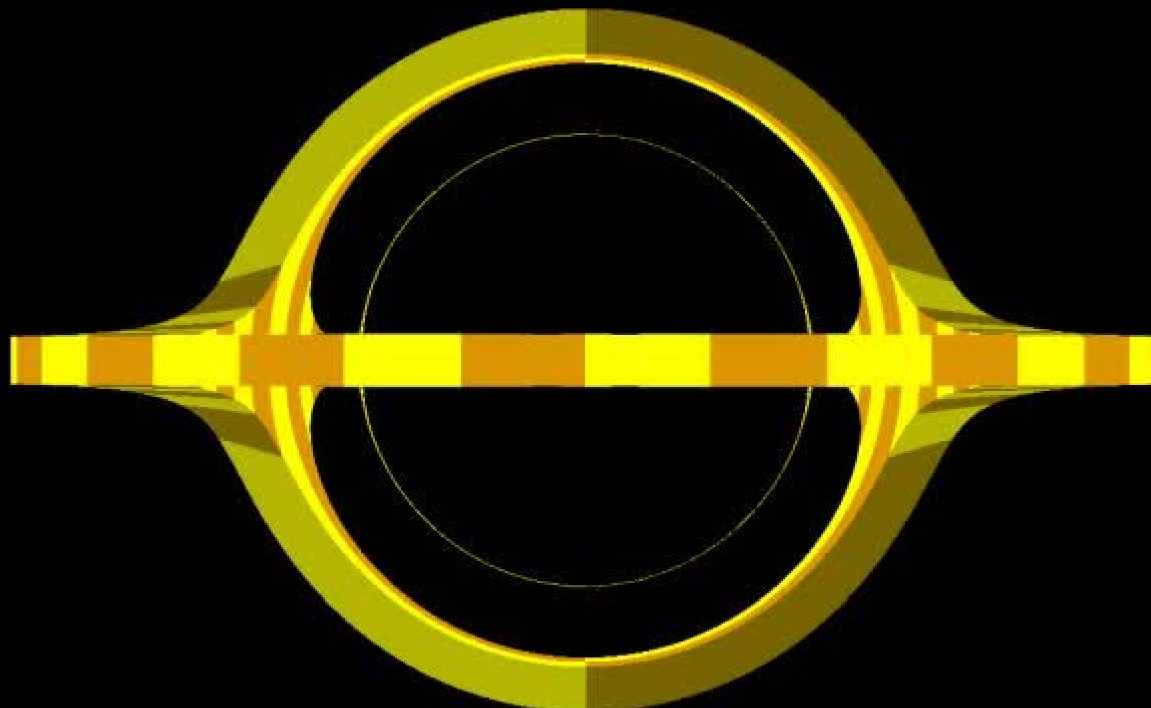

gravitational collapse (seen by a freely falling observer)
Corvin Zahn (1990), www.spacetimetravel.org



rotating ring around black hole

Corvin Zahn (1990), www.spacetimetravel.org

Physics 311
General Relativity

Lecture 14:
Gravity: Galileo, Newton, Einstein

Today’s lecture plan

- General Relativity: a theory of *gravitation*.
- Aristotle: “action is only by contact”
- Gravity of Galileo and Newton: a mysterious force.
- Einstein: *gravitation* is a property of space itself.

General Relativity – it's all about *gravitation*

• First and foremost, why *gravitation* rather than *gravity*? And is there any difference?

• Technically speaking, both "gravity" and "gravitation" refer to the same phenomenon: massive bodies moving towards one another.

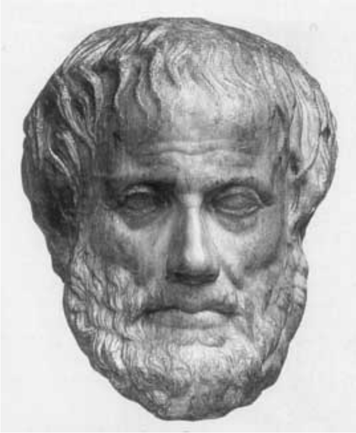
• "Gravity", however, was thought to be force mysteriously acting at a distance. Einstein did not accept action at a distance!

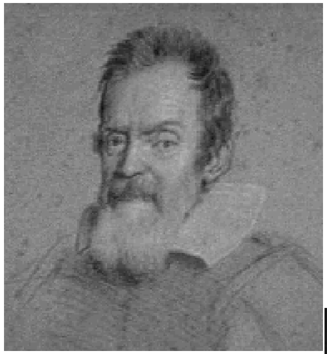
• So, he used the term "gravitation" to describe the *curvature of spacetime* caused by massive bodies. In General Relativity, and, indeed, in *Relativity in General*, there is no *local gravity*.

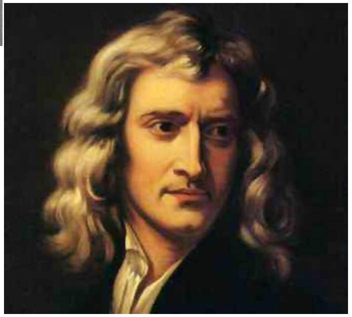
• Any free-fall (inertial) frame is local, no gravity exists in it! Effects of gravitation are only noticeable on non-local scale.

The history of (our understanding of) gravity

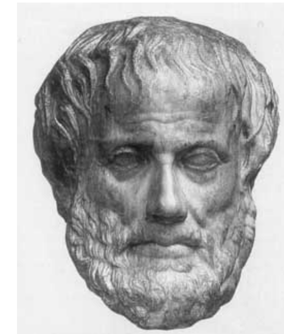
- Form Aristotle with his "fixed stars" and action by contact only – to Galileo and Newton with their *Æther* and action at the distance – to Einstein with his curved space.







Aristotle



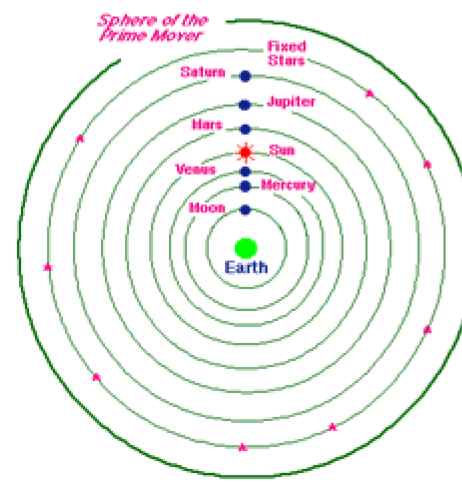
• Aristotle's views dominated science for centuries, in fact impeding the science development.

• Aristotle postulated that:

- action is only possible by direct contact
- constant force is required to maintain uniform motion

• So, if you hold an apple, you are applying a constant force to it – *by direct contact!* - and it remains in constant motion (rest). If you let apple go, it falls, *because there is no force applied to it*. Then it reaches the ground, *gets into the contact with Earth*, and resumes its uniform motion (rest).

• Thus, there's no gravity – when the apple is falling, there's no force acting on it. And, of course, the Sun, the Moon, the stars and planets are all attached to rigid spheres that rotate around the (flat) earth.

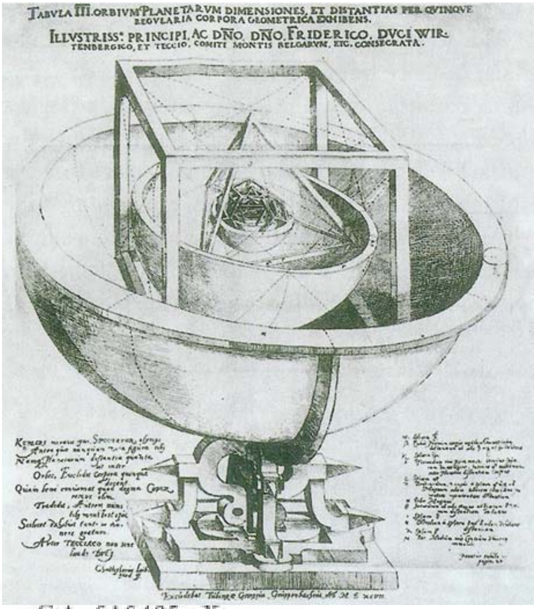


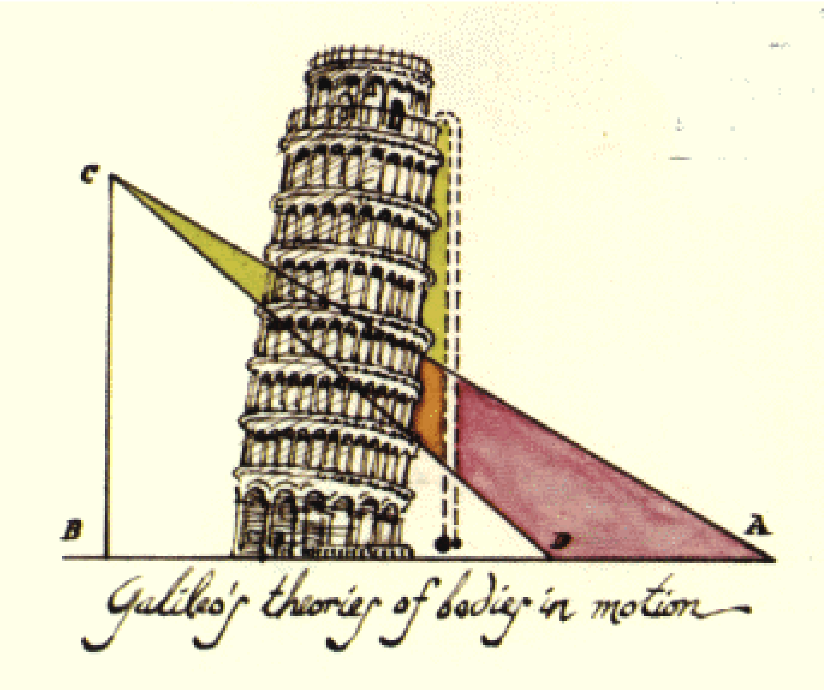
Aristotle's Universe

Galileo and Newton

• Kepler's laws (of the motion of celestial bodies) and Galileo's understanding of motion of falling bodies led Newton to discovering his theory of gravity. Newton's law of gravity (we all know and love):

$$F = G\, M_1 M_2 / R^2$$



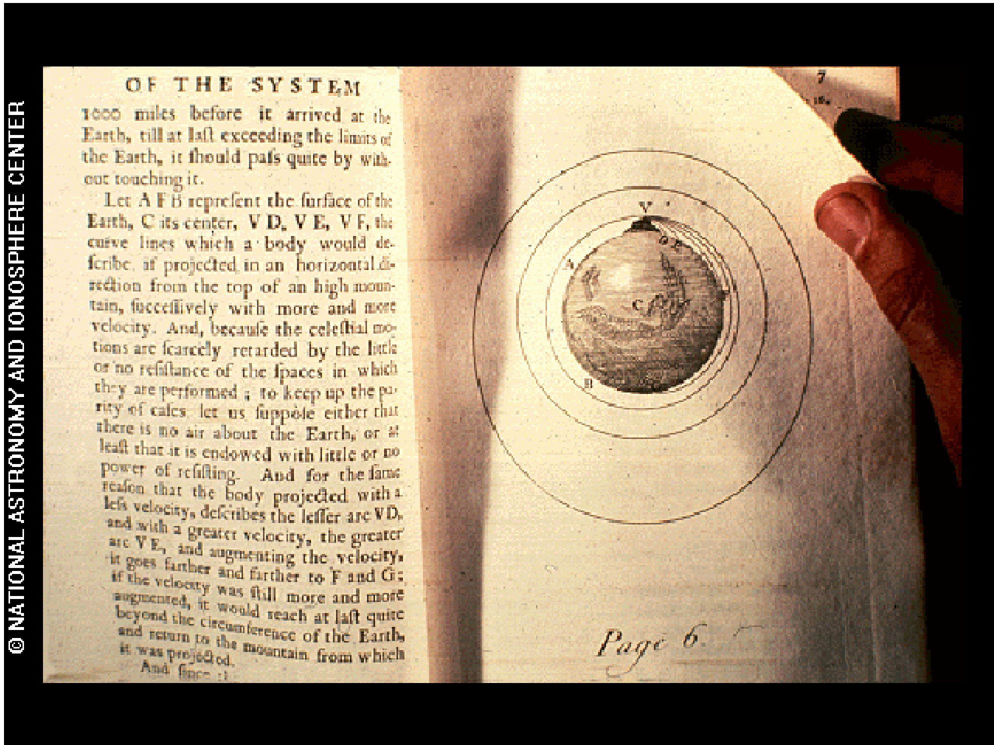


Newton’s gravity

• Newton’s gravity – the inverse-squares law, gravitational force dependence on the mass of the bodies – has been found very successful and in an excellent agreement with all measurements in the solar system, and on Earth.

• Lagrange and Hamilton developed very general methods to describe *gravitational fields* through *potentials* (integrals of the force). These potentials were used by Poisson to solve Newton’s equation in very general cases (not just point masses or spheres).

• One big question: where does this force come from? How do bodies ***know*** about each other? If I put an object inside a box, it still seems to know about the Earth’s presence...


OF THE SYSTEM

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1000 miles before it arrived at the Earth, till at last exceeding the limits of the Earth, it should pass quite by without touching it.

Let AFB represent the surface of the Earth, C its center, VD, VE, VF, the curve lines which a body would describe, if projected in an horizontal direction from the top of an high mountain, successively with more and more velocity. And, because the celestial motions are scarcely retarded by the little or no resistance of the spaces in which they are performed; to keep up the parity of cases let us suppose either that there is no air about the Earth, or at least that it is endowed with little or no power of resisting. And for the same reason that the body projected with a less velocity, describes the lesser arc VD, and with a greater velocity, the greater arc VE, and augmenting the velocity, it goes farther and farther to F and G; if the velocity was still more and more augmented, it would reach at last quite beyond the circumference of the Earth, and return to the mountain from which it was projected.

And since ...

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Rigid Euclidean Frame

• Newtonian mechanics happens with respect to a universal, rigid Euclidean reference frame. A free particle moves along a straight line with respect to that frame. Pull of the gravity deflects the free particle from this straight path. In fact, presence of the gravity *everywhere* in the Universe makes straight motion – with respect to the rigid frame – impossible.

• In the late 19th and early 20th century there was a growing discontent with this theory, especially its action-at-a-distance aspect. Lorentz and Poincaré in the early 1900 suggested that gravity should propagate at the same speed as light, and be in form of waves.

• We all know that the presence of a unique reference frame was challenged and rejected by Einstein's special relativity in 1905. However, it wasn't until 1907 that he began to think about incorporating gravity into his relativity theory...

Einstein’s quest to General Relativity

- In 1907, while preparing a review of his Special Relativity, Einstein began to wonder how Newtonian gravity would have to be modified to fit in with Special Relativity. (Obviously, since Newtonian gravity was based on a unique reference frame, it must be wrong!)

- It was then that Einstein proposed his ***equivalence principle*** – that “*... we shall therefore assume the* ***complete physical equivalence*** *of a* ***gravitational field*** *and the* ***corresponding acceleration of the reference frame****. This assumption* ***extends the principle of relativity to the case of uniformly accelerated motion*** *of the reference frame.*”

- This conjecture was based on one of Einstein happy thoughts – that an observer that’s fallen off the roof of a house experiences no gravitational field!

Astronomical observations

• Einstein realized that the equivalence principle implies bending of light in the vicinity of massive bodies. Since gravitation and acceleration are the same, then observing light as it passes near a large mass is equivalent to the observer accelerating, which leads to the light path to appear curved.

• In 1911 Einstein proposed an astronomical method to measure the deflection of light: measuring positions of stars as Sun passes in front of them – during the solar eclipse, so that stars can be seen.

The Einsteinturm – "Einstein Tower" built in Potsdam around 1920 to measure the deflection of star light by the Sun

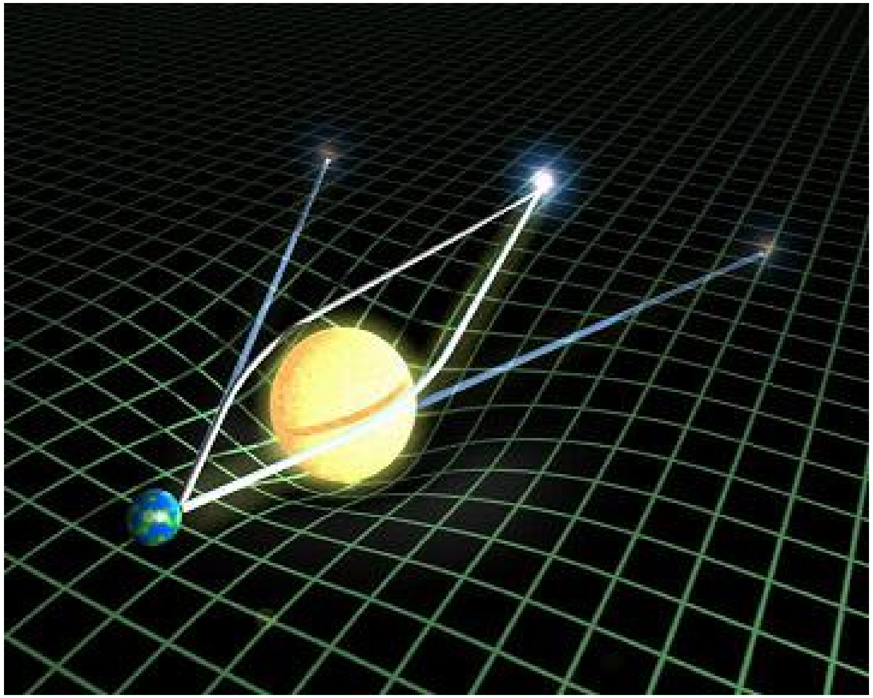
Einstein’s blunder and curved space

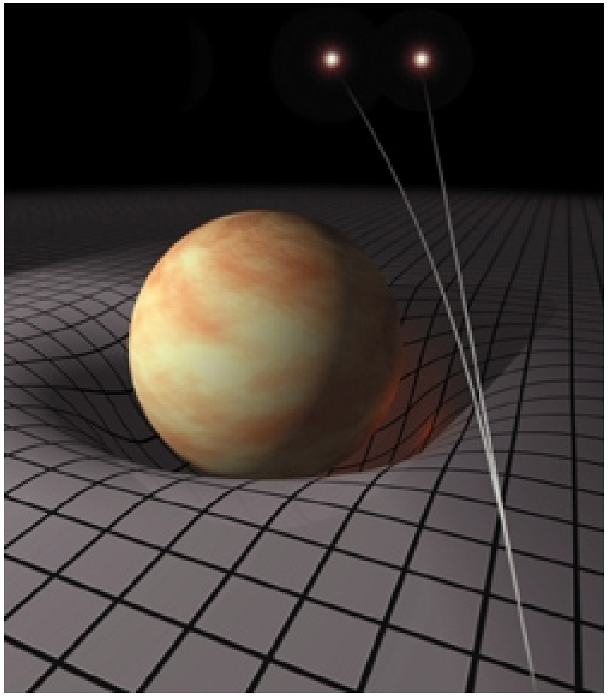
• However... In 1911 Einstein’s calculations for bending of light there was a factor of 2 error! Good thing he caught the error himself – in 1915.

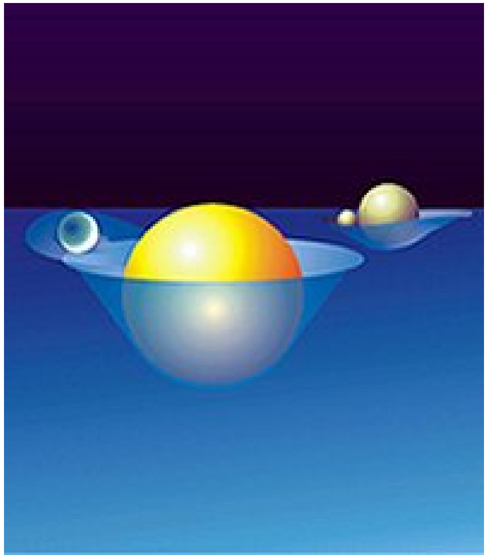
• 1915 was the year when Einstein (and, almost simultaneously, Hilbert) realized that if he were to postulate the equivalence of all ***accelerating*** frames, then space could not be Euclidean!

• Recall: the equivalence of all ***inertial*** (i.e. non-accelerating) frames was the postulate of Special Relativity. The postulate of equivalence of all ***accelerating*** or ***non-inertial*** frames became central in General Relativity.

• Thus, Einstein adapted the notion of curved space (and time!), and postulated that gravity is not a physical force. Rather, curvature of spacetime leads to masses wanting to move towards each other.





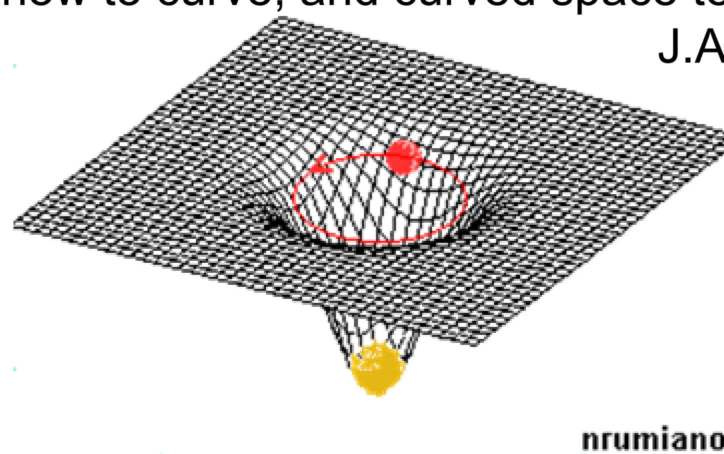


The Field Equation

- To take into account spacetime curvature, tensor calculus was necessary in Einstein's General Relativity.

"matter tells spacetime how to curve, and curved space tells matter how to move"

J.A. Wheeler



Dissecting the Field Equation

- The full form of the field equation is: $G_{\mu\nu}$

$$R_{\mu\nu} - ½Rg_{\mu\nu} = (8\pi G/c^4)\ T_{\mu\nu}$$

- What are these $G_{\mu\nu}$ and $T_{\mu\nu}$? They are:

  - $G_{\mu\nu}$ is *Einstein tensor*, it is equal to $[R_{\mu\nu} - 1/2Rg_{\mu\nu}]$ - the *Ricci tensor* minus ½ *Ricci scalar* times *metric tensor,*

  - $T_{\mu\nu}$ is the stress-energy tensor,

  - G is the gravitational constant (which is taken to be 1 in the short form of the field equation),

  - c is speed of light (which we take to be 1), and

  - π is simply the number π. We do not take it to be 1.

- Each of these tensors is a symmetric 4x4 tensor – thus, only 10 components are independent... Why?

Symmetric 4x4 tensors

• Let's write a 4x4 tensor as a matrix:

$$\begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{pmatrix}$$

• The tensor is called symmetric if it does not change under transposing of the matrix, i.e. when $A_{nm} \rightarrow A_{mn}$, thus $A_{nm} = A_{mn}$.

• This means that all tensor components above the diagonal have equal counterparts under the diagonal, and there is only

$$16 – 6 = 10$$

independent components!

Symmetric 4x4 tensors – part 2

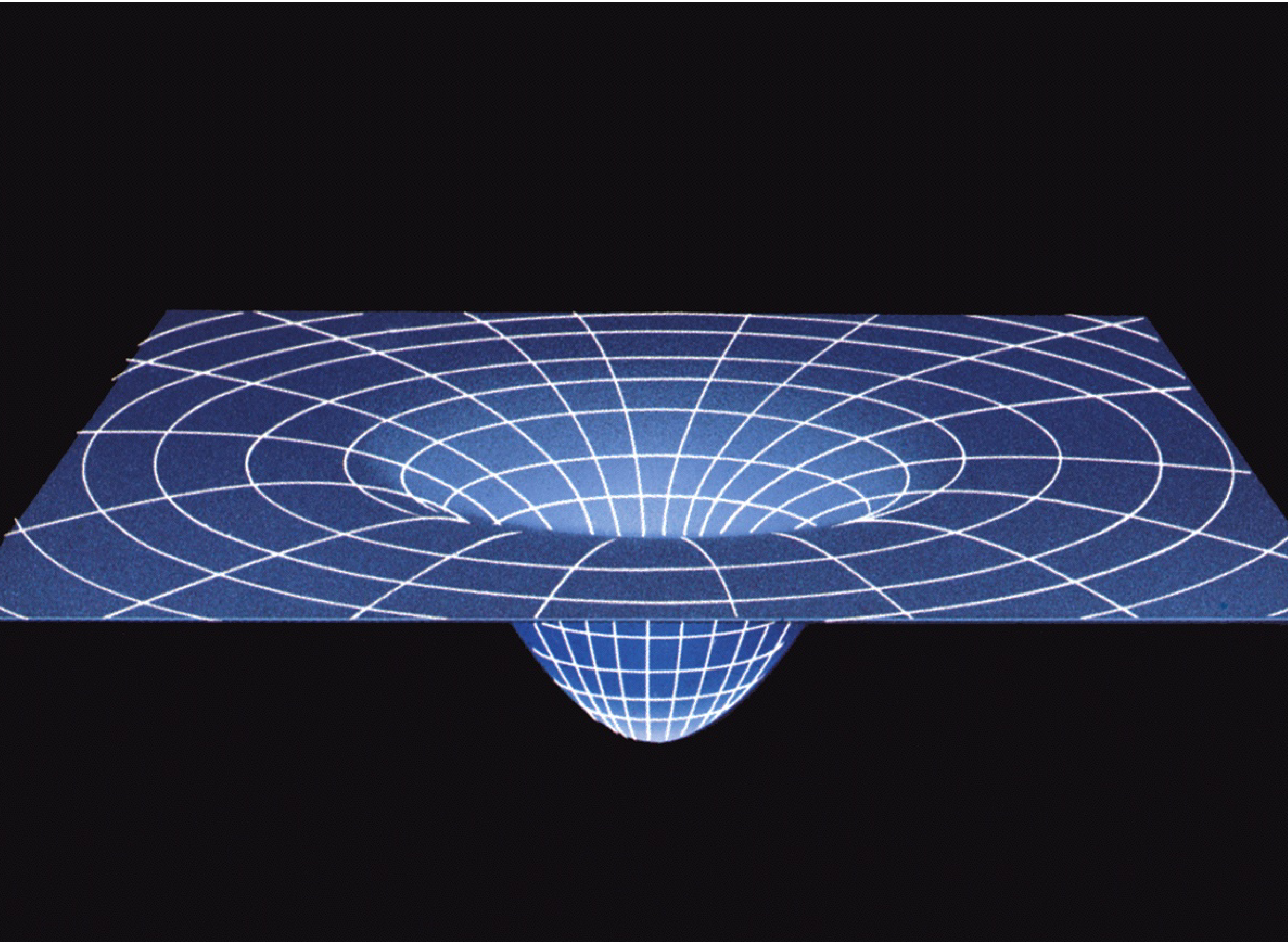
- But it gets even better! If we can choose 4 spacetime coordinates (which we can – {t, x, y, z} is one possibility), then total number of independent equations reduces to 10 - 4 = 6.
- Isn't that great?! Six equations is all you need to solve, and *voila* – you have your gravity and everything! (Well, what you really do is you solve for the metric tensor $g_{\mu\nu}$ given distribution of masses, that defines the stress-energy tensor $T_{\mu\nu}$ - not a trivial thing).
- Only 6 equations indeed, but (in general) very-very hard equations. Only few special cases have been solved analytically – we'll see some solutions, including the Schwarzschild metric – in our future lectures.
- Numerical Relativity is a branch of physics of its own whose only purpose is to solve the Einstein Field Equation numerically for realistic conditions.

Correspondence principle

• Remember: the new theory must agree with the old theory where the old theory gives correct predictions. This is called "correspondence principle" and we have seen it applied to Special Relativity – relativistic velocity addition was well approximated by Galilean velocity addition for small velocities, Lorentz transformations corresponded to Galilean (with small correction) at small velocities...

• So, one should expect that equations of General Relativity should simplify to Newtonian gravity for weak fields (low gravity – low spacetime curvature – almost flat (Euclidean) space – Newtonian gravity!), or small velocities.

• There's correspondence, indeed, but we'll have to revisit this topic later, when we are brave enough to solve the Einstein field equation, to show this correspondence.



Recap:

- Gravitation of General Relativity is not a force; it is the property of spacetime, its curvature.
- Mass curves spacetime, while curved spacetime determines the motion of mass.
- Mathematical form of General Relativity is the Einstein's Field Equation that binds together the metric (curvature) of spacetime and the stress-energy (mass distribution).
- General solution of Einstein's Field Equation is difficult; some important special-case solutions are known.