Wednesday, October 24:

Midterm #1

Material covered:

- Events, reference frames, spacetime
- The interval
- Simultaneity (and **relativity thereof**)
- Lorentz transformations
- Twin Paradox

Closed book. May bring a single formula sheet.

Physics 311
Special Relativity

Lecture 9:

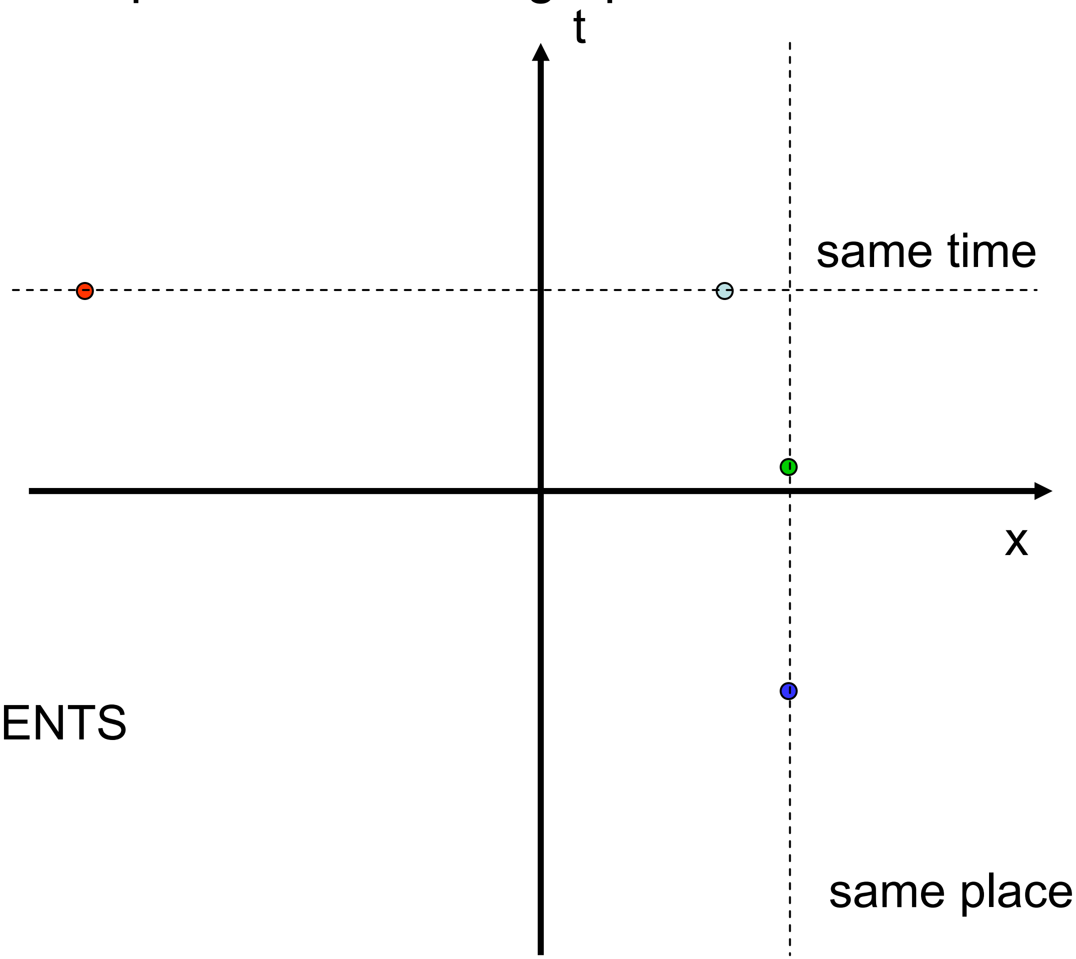
Worldline. Light cone.

Today’s lecture plan

- Spacetime maps. Events in different spacetime maps.
- Invariant hyperbola
- Worldline. Length and time. Maximal aging.
- Spacelike, timelike and lightlike intervals.
- Light cone: partition of spacetime.

Spacetime maps

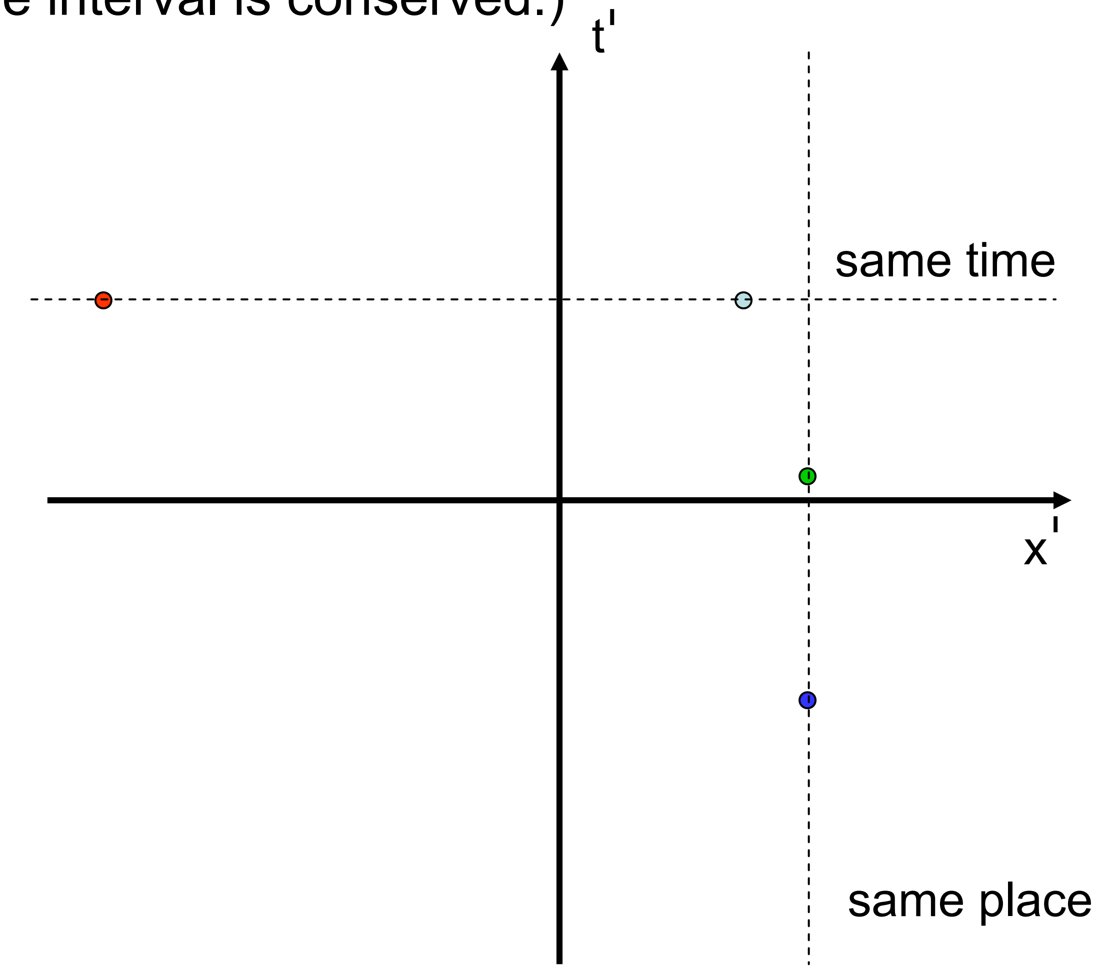
• We've seen them before: space on the horizontal axis, time on the vertical axis. Not a position vs. time graph! Because of what we plot...



We plot EVENTS

Spacetime maps are different in different frames

• Space and time coordinates of events depend on the reference frame. (Although the interval is conserved.)



Invariant hyperbola

- Consider two events:

  Event 1: $x = x' = 0, t = t' = 0$ (reference)
  Event 2: $x = x_2$, $x' = x_2'$, $t = t_2$, $t' = t_2'$

- The interval:

$$s^2 = t_2^2 - x_2^2 = (t_2')^2 - (x_2')^2$$

- What geometric shape does this expression define?.. Some kind of second-order line...

  Circle?... $x^2 + y^2 = R^2$ ... NO!...

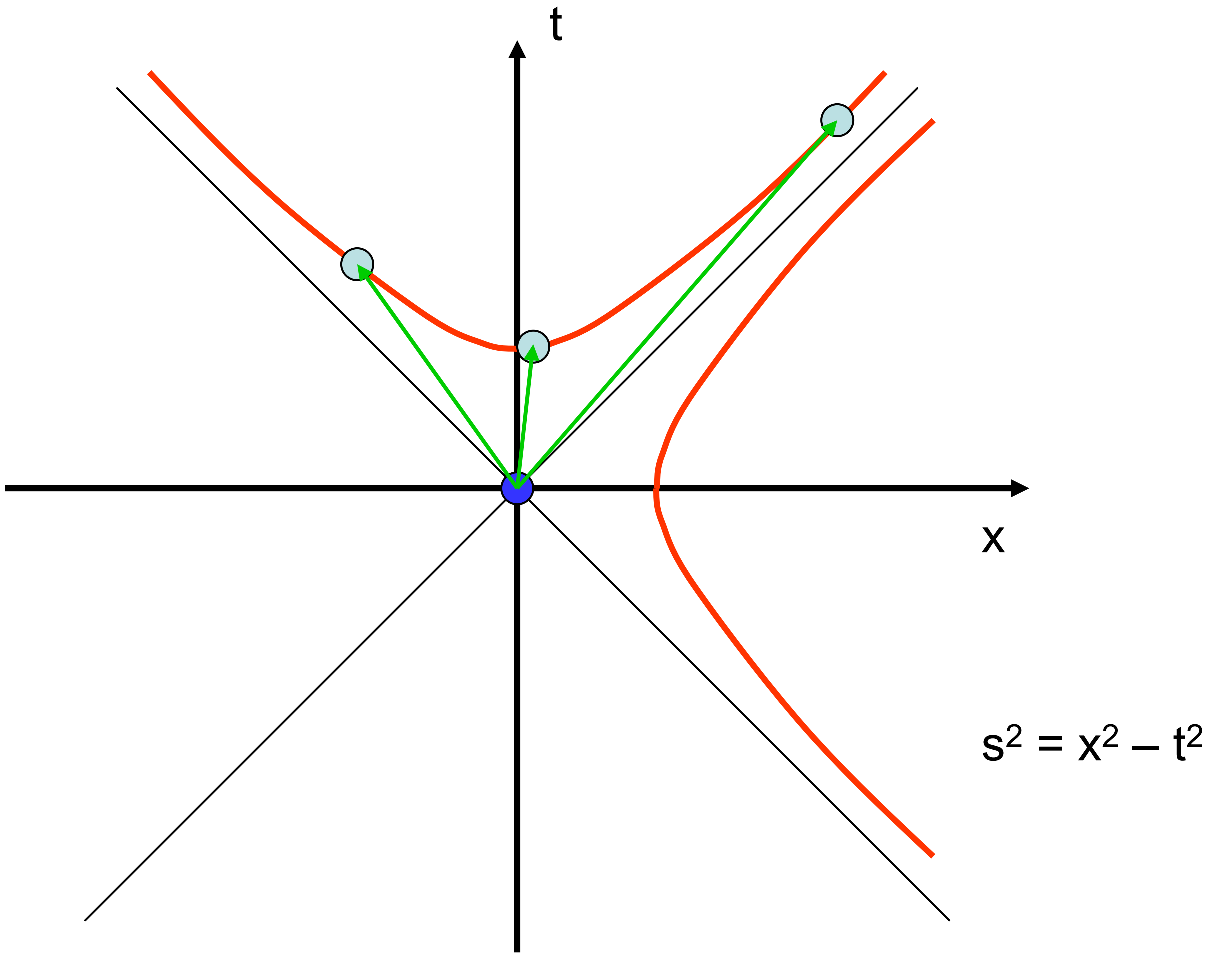
  Ellipse?... $x^2/a^2 + y^2/b^2 = 1$ ... NO!...

  Hyperbola! $x^2 - y^2 = c^2$ YES!

(In fact here we have a hyperbola of the form $y^2 - x^2 = c^2$, with its opening along the vertical axis)

“Events live on a hyperbola”

- Observers in all frames agree that events lie somewhere on the same hyperbola.



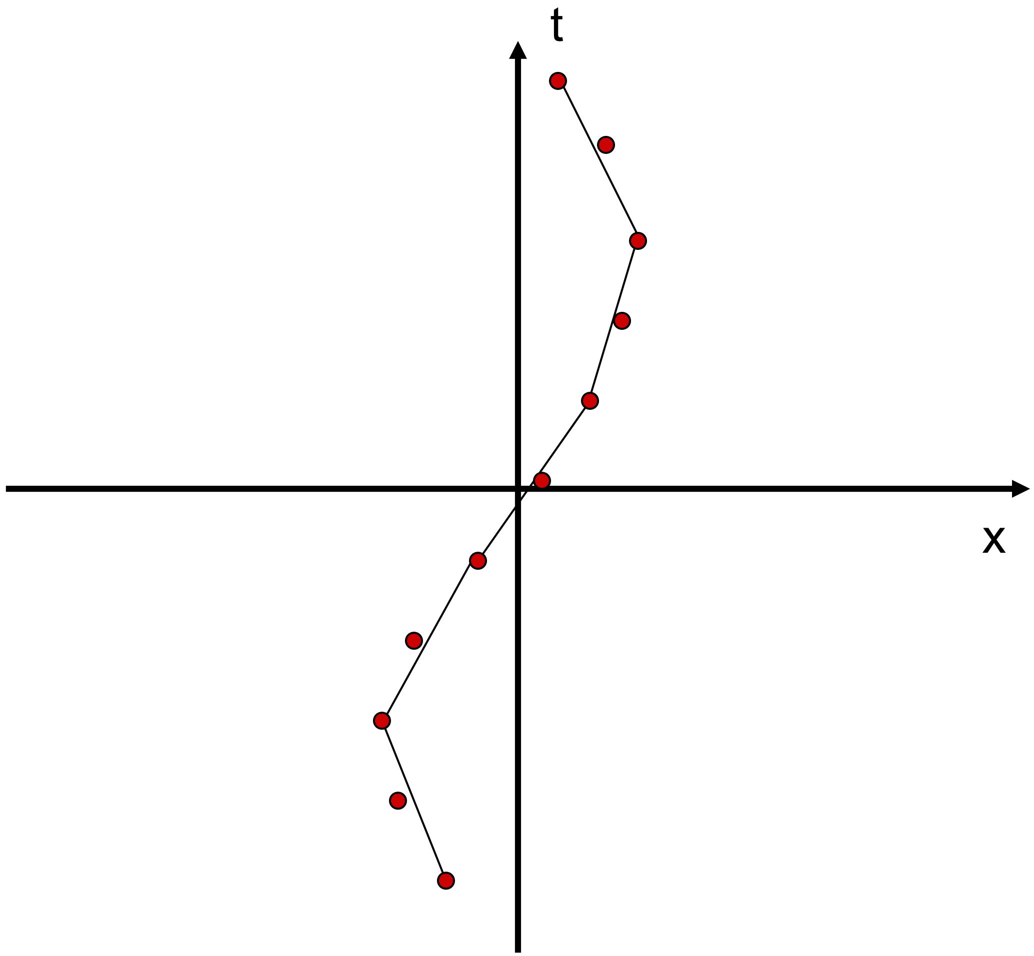
Moving particle

• Let's observe a particle moving through our frame of reference. We'll measure the particle's position in space every second. To each such measurement we assign an event with time corresponding to the time of the measurement and position corresponding to the position of the particle. (Conversely, we could use the latticework of clocks and record times when the particle flies close to a clock, then assign events based on the measured times and positions of the respective clocks).

• When we plot these events on the spacetime map, a line is traced...

... the worldline!

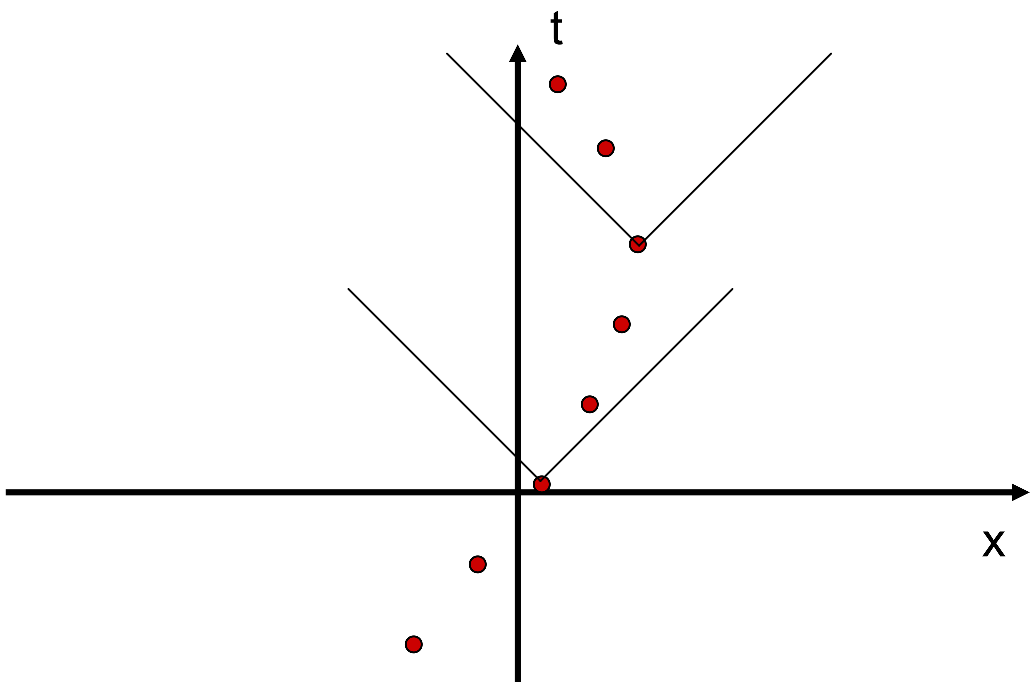
• The events are like pearls on a string, and the string is the worldline. Add more pearls, make the line more smooth...



Particle speed

• A particle moving at constant speed will leave a straight line trace on the spacetime map. Its worldline is straight. The (inverse) slope of the line is the speed (because we plot time on the vertical axis). I will use the word "slope" meaning "slope" and not "inverse slope". Then particle with *higher* speed will have *smaller* slope. A particle at rest will have infinite slope.

• Maximum speed – the speed of light c = 1 has the slope of 1. Nothing moves faster than light, so worldline slope can not be smaller than 1.



Changing speed – curving worldline

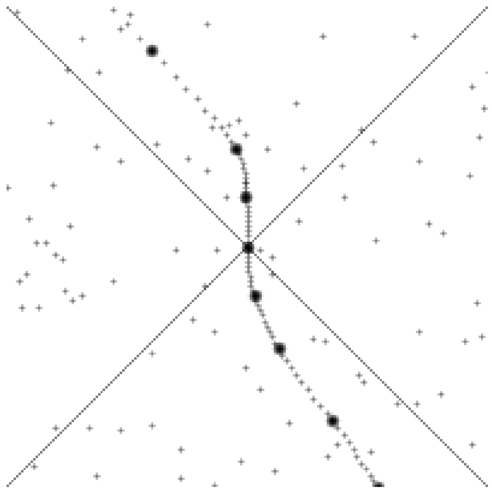
• Since the slope of the worldline is the inverse of the particle's speed, then when the speed changes, the worldline curves.

• This does not mean that the particle's direction of motion changes! Remember, we are looking at a one-dimensional motion, motion along the x-axis. A train moving along straight tracks. A rocket flying along a straight line to Canopus.

• The worldline gives a complete description of particle's motion in spacetime: its coordinates give the particle's position in space and time, its slope gives the particle's speed.

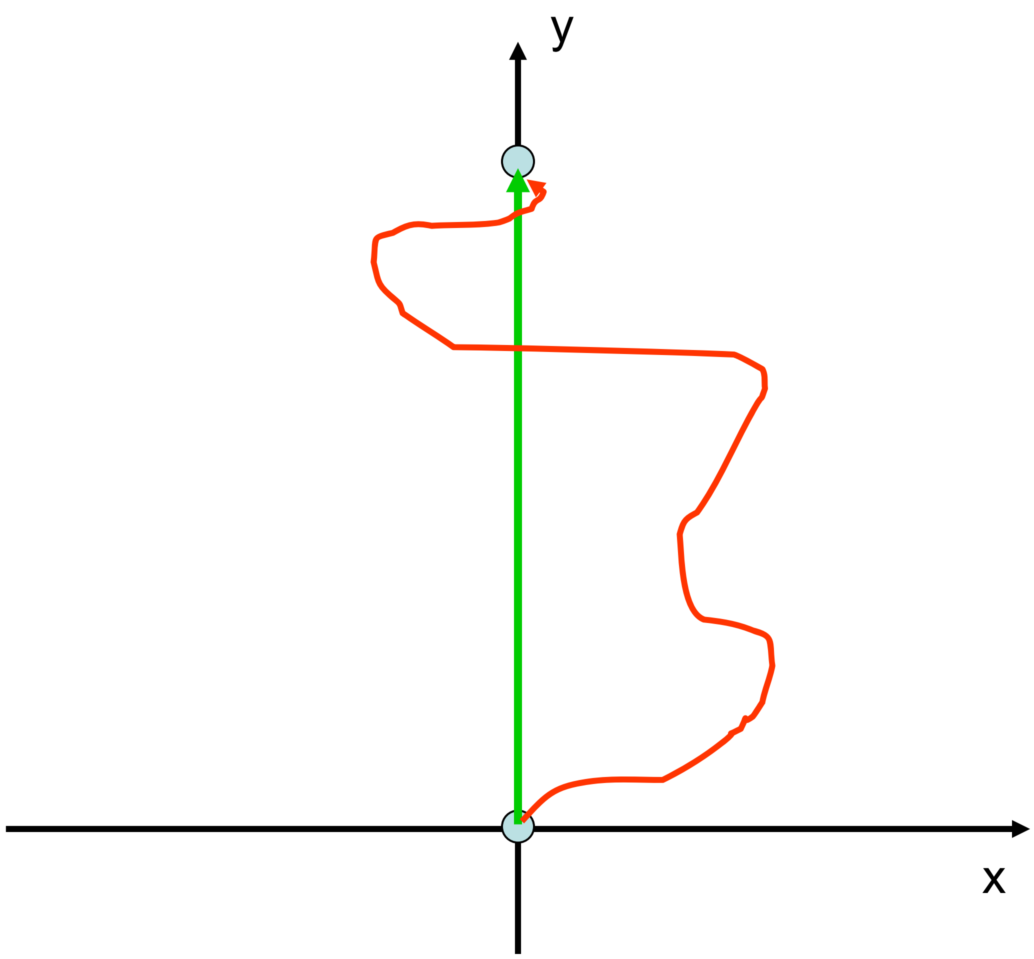
• However, the worldline can only describe the events that have already happen. Worldline does not predict the future; it is not an equation of motion!

Animated worldline – from Wikipedia



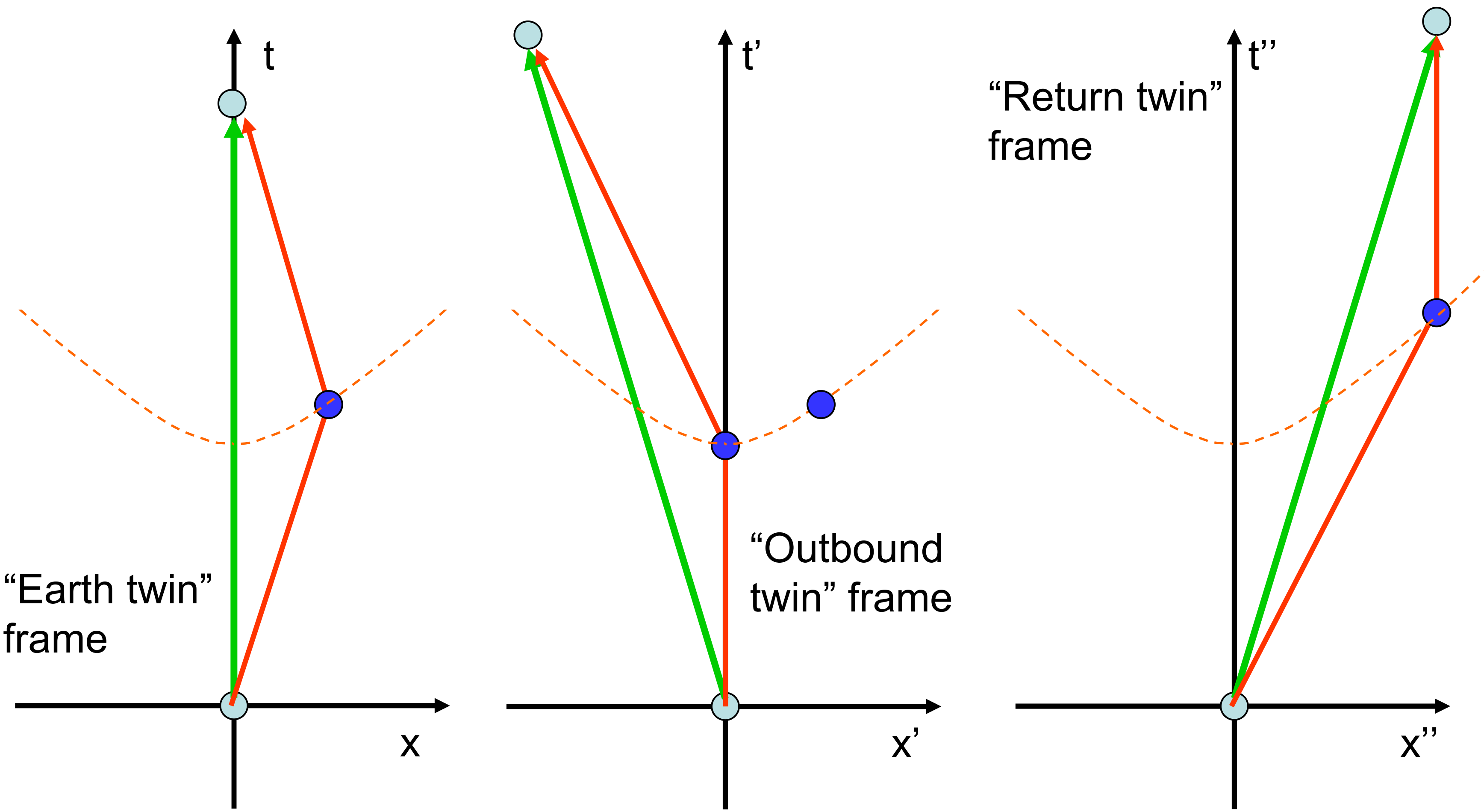
Straight line in space: the shortest distance

- A straight line connecting two points is the shortest path in Euclidean geometry. It is so obvious! Do I even have to prove it?



Straight line in spacetime: the longest proper time

• In Lorentz geometry, the straight line connecting two events corresponds to the frame with the *longest* proper time! Recall the Twin Paradox:



Proper time and the Principle of Maximal Aging

- Proper time is accumulative. Elapsed proper time is actually equivalent to the aging of the observer at rest in the frame of interest.
- A straight line in spacetime map represents a particle moving at a constant speed – a *free particle*.
- Principle of Maximal Aging: a free particle follows a straight path between events. Straight path has the longest proper time. Thus, a free particle follows the worldline of maximal aging.
- Principle of Maximal Aging also works in curved spacetime of General Relativity, where a free particle follows a curved path in spacetime.

Speed of light: the limit of causality

• Objects moving at speed of light (photon is the only candidate at this point, the graviton, if discovered, would be the other, neutrinos are out...) trace a 45-degree worldline on the spacetime map. No object can travel faster. Nor can information.

• Two events can be *causally related* if information about Event 1 can be transmitted to the location of Event 2 before the Event 2 occurs. Then one can say that Event 1 was the *cause* of Event 2.

• The interval between causally related events must have “more or equal time than space” – there must be enough time to cover the space at speed of light or less.

• When USS Enterprise goes at Warp-9, their arrival at destination cannot be caused by their departure from the starting point!

Intervals: timelike, spacelike, lightlike

• Depending on ratio of time and space parts in the interval between two events, intervals are divided into:

- *timelike* – the time dominates, interval^2 = $\Delta t^2 - \Delta x^2$

- *spacelike* – the space dominates, interval^2 = $\Delta x^2 - \Delta t^2$

- *lightlike* – space and time are equal, interval^2 = 0

• Intervals of different kind live in different regions of spacetime, they never visit each other's domains.

• Timelike intervals live on a hyperbola that opens along the time axis.

• Spacelike intervals live on a hyperbola that opens along the space axis.

• Lightlike intervals live on the 45-degree line. They are special.

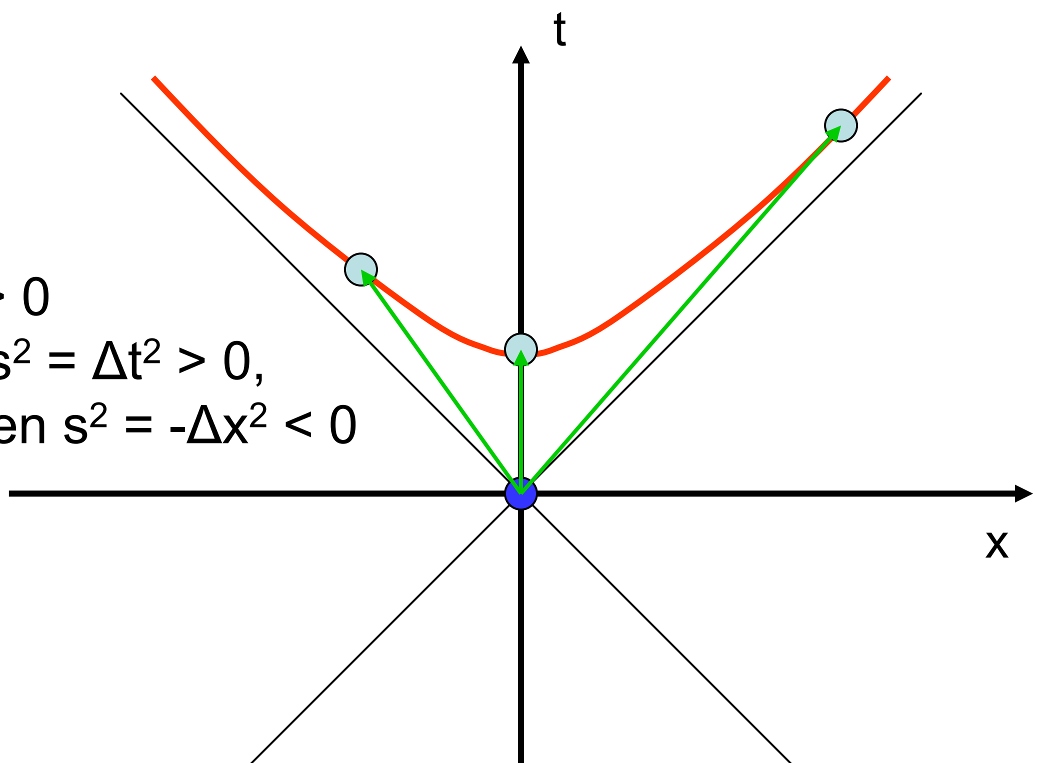
Timelike interval

- An interval between causally-related events.
- You can show that there exists a frame where the two events happen in the same place.
- You can also show that there's no frame where events happen at the same time.

$s^2 = \Delta t^2 - \Delta x^2 > 0$

if $\Delta x = 0$, then $s^2 = \Delta t^2 > 0$,

but if $\Delta t = 0$, then $s^2 = -\Delta x^2 < 0$



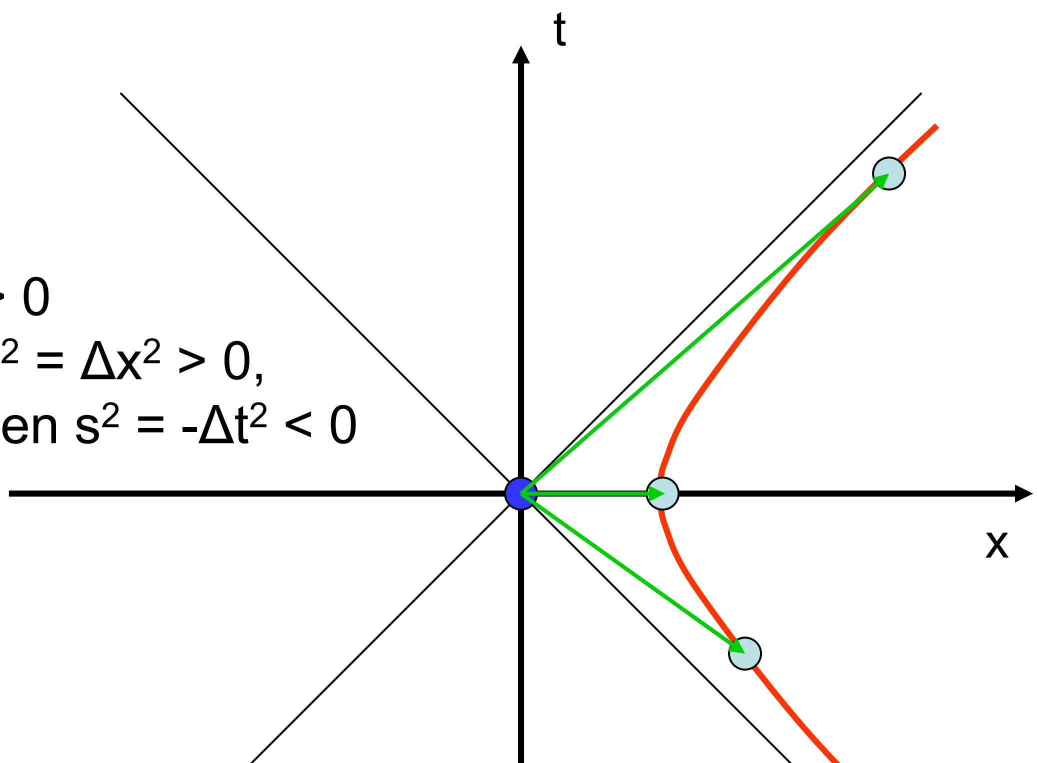
Spacelike interval

- An interval between causally-unrelated events.
- You can show that there exists a frame where the two events happen at the same time.
- You can also show that there's no frame where events happen in the same place.

$s^2 = \Delta x^2 - \Delta t^2 > 0$

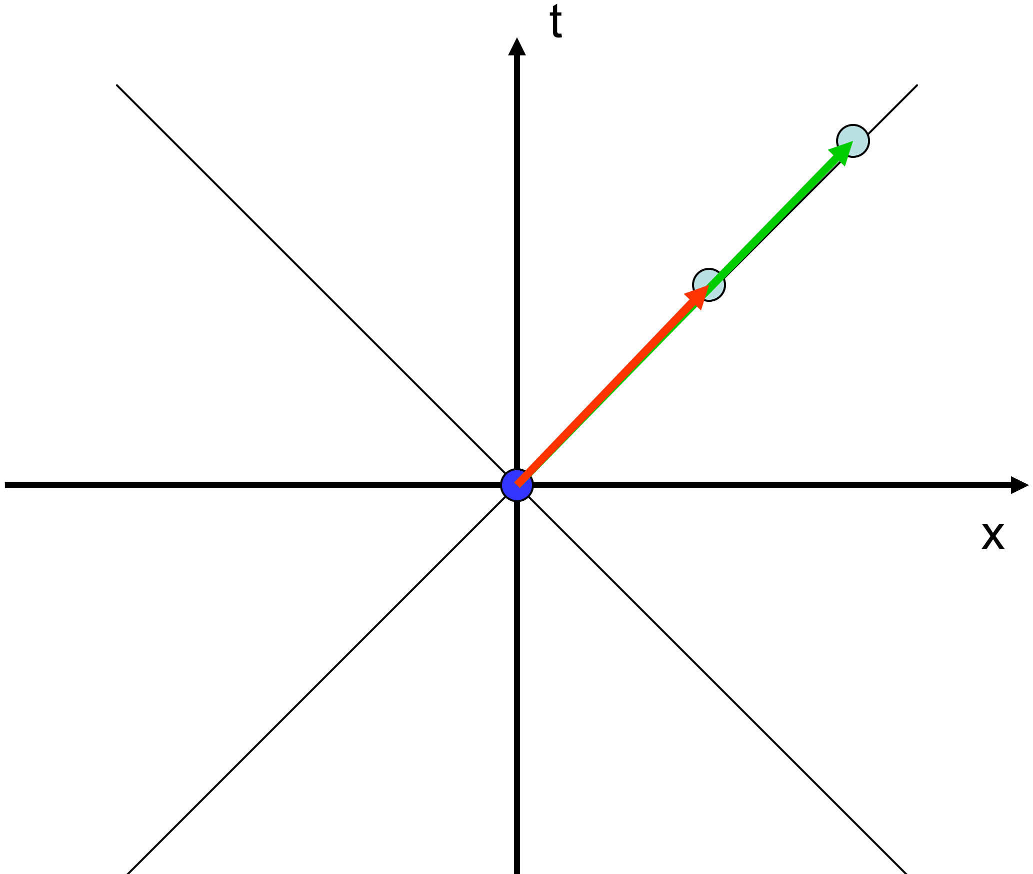
if $\Delta t = 0$, then $s^2 = \Delta x^2 > 0$,

but if $\Delta x = 0$, then $s^2 = -\Delta t^2 < 0$



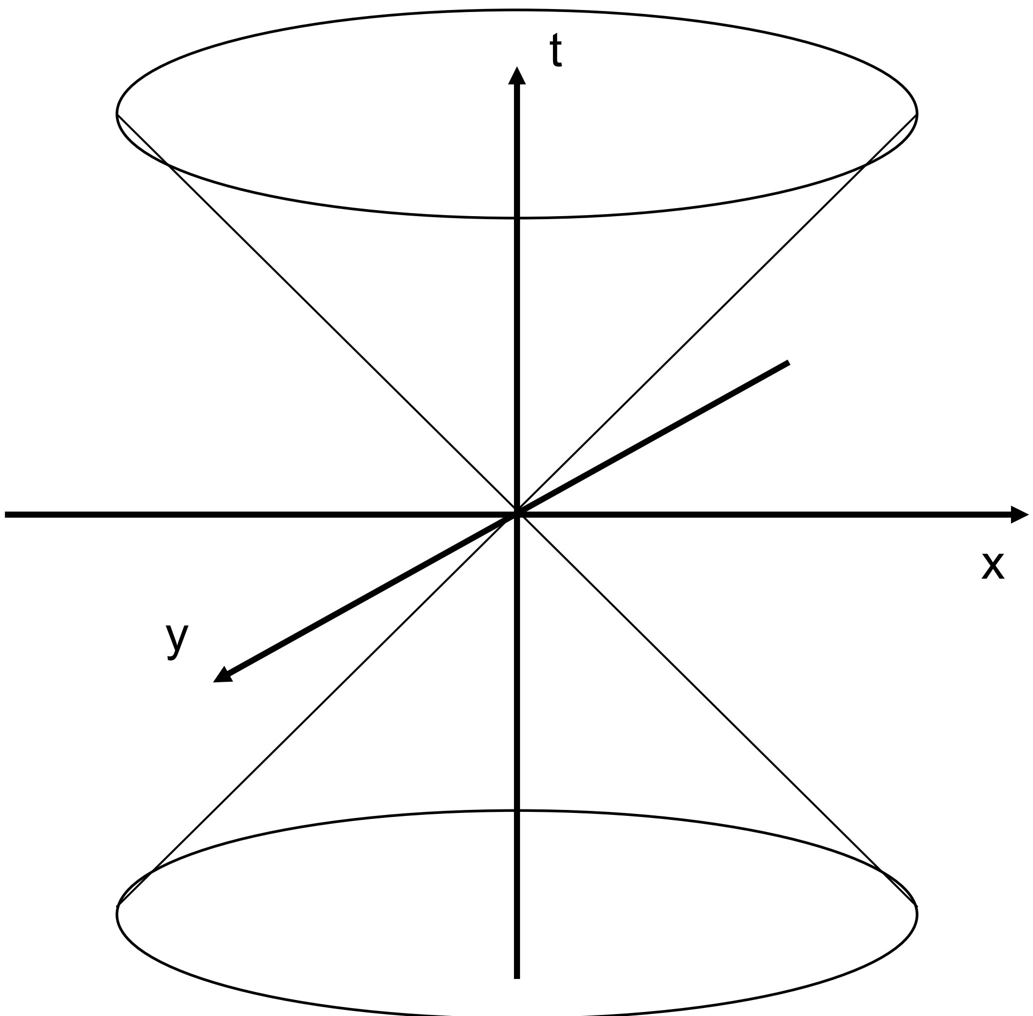
Lightlike interval

- Events can be causally-related – the information travels at speed of light.
- The interval is zero in all frames. Space and time parts can change between frames in relative motion, but remain equal to each other.



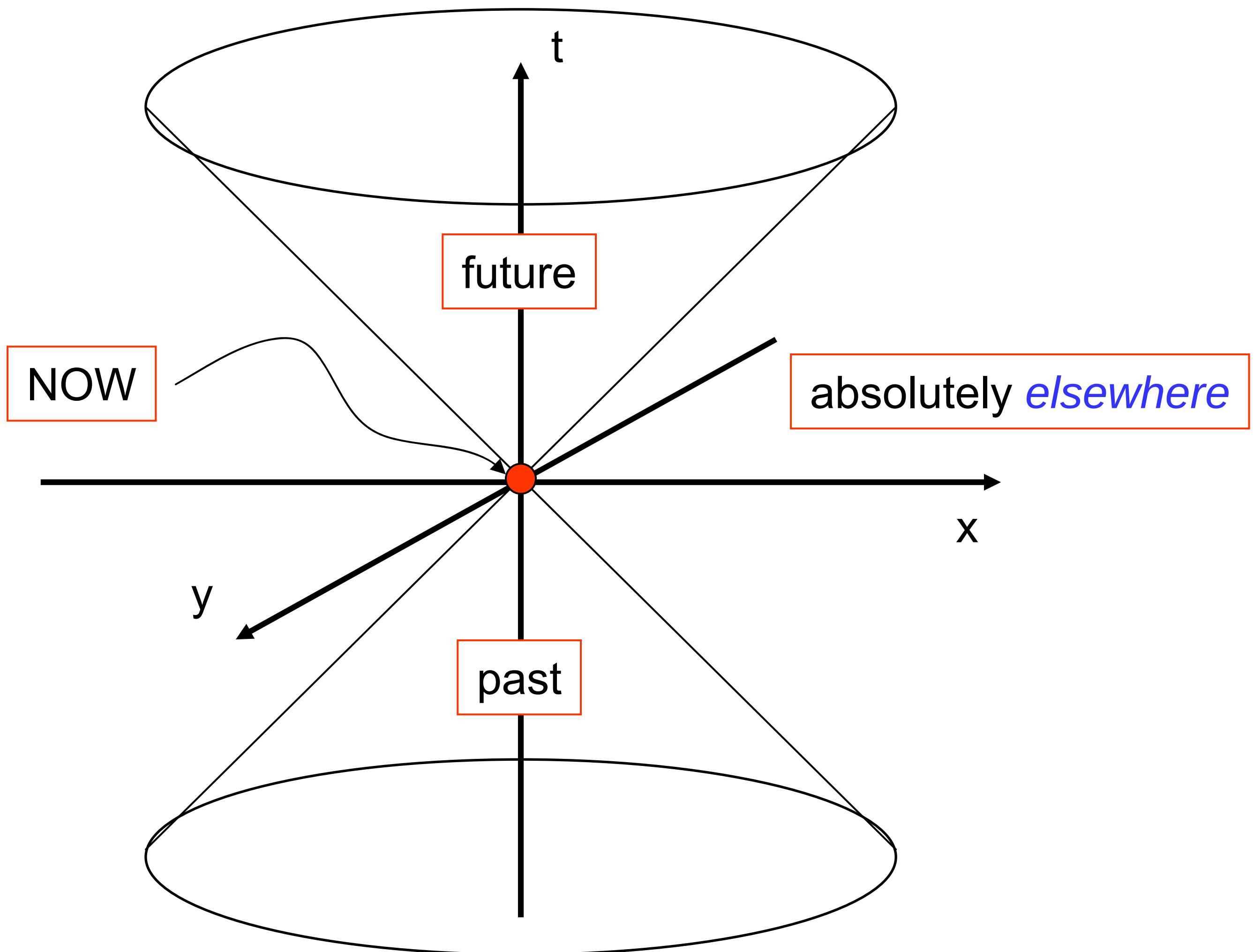
Light cone – partition of spacetime

- Why "cone"?..



Light cone – partition of spacetime

- Why "partition"?..



Partitions of spacetime

• Light cone preserves cause and effect. If Event 2 was caused by Event 1 in some reference frame, it will be also true in *any* frame. Event 2 will always stay within the light cone of Event 1.

• Positions at which causally-related Events 1 and 2 happen within the light cone varies from frame to frame. The events may even happen in the same spot.

• If no causal relation between Events 1 and 2 is possible (spacelike interval), the Event 2 will lie outside of the light cone of Event 1 in *any* reference frame. It is absolutely elsewhere.

• The relative time between causally-unrelated Events 1 and 2 can change from frame to frame. These events can be simultaneous in some frame.

• Events lying on the light cone stay on the light cone.