Homework 1

Due Sat. October 16 at 3:00 p.m. (on line)
or at the start of the Mon. October 18 lecture (on paper).

In BIOEN 301 you learned how to relate the deflection of a beam to the force applied along that beam. One of the simplest cases is a cantilevered beam with square cross section, with shear force applied at the tip, assuming the deflections are small. Because the ratio of force to deflection is one measure of a spring constant, you can use this ratio to write a differential equation that models the oscillatory behavior of the beam.

The beam in this problem is made of PDMS (poly-dimethyl siloxane), which is commonly used in bioengineering research because of its clarity, biocompatibility, and ability to replicate tiny features on a surface. One end is cantilevered, and at the other end is a steel bead. The beam is allowed to deflect horizontally, but not vertically, so we can ignore the effect of gravity. We will also ignore the mass of the beam, which is a poor assumption but let's keep things simple.

Relevant values are listed below. According to online sources,[1] Young's modulus, Y_{PDMS}, (also called elastic modulus, E), ranges from about 400,000 to 800,000 kPa; the value depends on the ratio of the base and activator used to make it. We'll assume a value in that range. The given damping constant was chosen to make ζ**,** the ratio of decay rate to oscillation rate, equal to 0.4. Note that the *effective* beam length is 3 cm.

<u>Relevant values</u>

Beam length	2.8 cm	Density of steel	7900 kg/m^3
Beam height	2 mm	Y_{PDMS}	700,000 kPa
Beam width	1 mm	Damping const.	7810 N-s/m
Bead diameter	4 mm	ζ (zeta)	0.4

a) Define $y(t)$ as the lateral deflection of the beam tip, and write a differential equation that represents the forced response of the beam+bead system.

b) Find the system's natural (unforced) response using any appropriate method. Let $y(0)$=1 mm and $y(0)'$=0.

c) Use Laplace transforms to find the system's response to $u(t)$ mm.

d) Use Laplace transforms to find system's response to $\delta(t)$ mm-sec.

e) Plot these three transients on the same graph, using MATLAB or Excel.

f) State what physical situation each of these inputs represents.

g) Find the derivative of the step response and compare it to the impulse response.

h) Round beams are commonly used in PDMS structures such as these. Determine the diameter of a round beam that has the same response characteristics as the structure described above.

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[1] http://web.mit.edu/6.777/www/matprops/pdms.htm