SOLUTION FOR HW #3

$\dot{D} = (30-W)D \quad , \quad \dot{W} = (.5D-40)W + f$

1. Equilibrium points
Set derivatives and input to zero

$\dot{D} = 0 = (30-W)D \rightarrow$ a) $W = 30$ b) $D = 0$

a) $\dot{W} = 0 = (\frac{D}{2} - 40)30$, $D = 80$

b) $\dot{D} = 0 = (\frac{0}{2} - 40)W$, $W = 0$, $(0,0)$

Two points $\rightarrow (80,30)$

2. System is linearized using Jacobian matrix, which is 2-D version of Taylor series

$$\begin{bmatrix} \frac{\partial \dot{D}}{\partial D} & \frac{\partial \dot{D}}{\partial W} \\ \frac{\partial \dot{W}}{\partial D} & \frac{\partial \dot{W}}{\partial W} \end{bmatrix} = \begin{bmatrix} 30-W & -D \\ .5W & .5D-40 \end{bmatrix}$$

Evaluate at $(0,0)$:

$$A_{0,0} = \begin{bmatrix} 30 & 0 \\ 0 & -40 \end{bmatrix}$$

Evaluate at $(80,30)$:

$$A_{80,30} = \begin{bmatrix} 0 & -30 \\ 40 & 0 \end{bmatrix}$$

3. Write system in state space form near each eq. point.

Near $(0,0)$: $\begin{bmatrix} \dot{D} \\ \dot{W} \end{bmatrix} = \begin{bmatrix} 30 & 0 \\ 0 & -40 \end{bmatrix} \begin{bmatrix} D \\ W \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} f \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} D \\ W \end{bmatrix}$

Near $(80,30)$: $\begin{bmatrix} \dot{D} \\ \dot{W} \end{bmatrix} = \begin{bmatrix} 0 & -30 \\ 40 & 0 \end{bmatrix} \begin{bmatrix} D_e \\ W_e \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} f \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} D \\ W \end{bmatrix}$

NOTE 1: I have chosen the output y to be deer — you will be given this information on the quiz.

Note 2: The new variables $D_e = D - 80$ and $W_e = W - 30$ are used near the point $(80,30)$

4. Eigenvalues

Near $(0,0)$ $\quad |sI-A| = \begin{vmatrix} s-30 & 0 \\ 0 & s+40 \end{vmatrix} = 0$

$$(s-30)(s+40) = 0$$

The eigenvalues are 30 and -40.

#5. This system is unstable, with one pole in the right half plane.
It produces a saddle point because the roots are real and have opposite sign.

Near $(80,30)$ $\quad |sI-A| = \begin{vmatrix} s & 30 \\ -40 & s \end{vmatrix} = 0$

$$s^2 + 1200 = 0$$

Eigenvalues $\longrightarrow$ $s = \pm j\,20\sqrt{3}$

This system is conditionally stable, with two poles on the imaginary s-axis.
It produces a center near $(80,30)$.

6. Sketch phase portrait.
Find $\dot{D}$ and $\dot{w}$ near the eq. points.

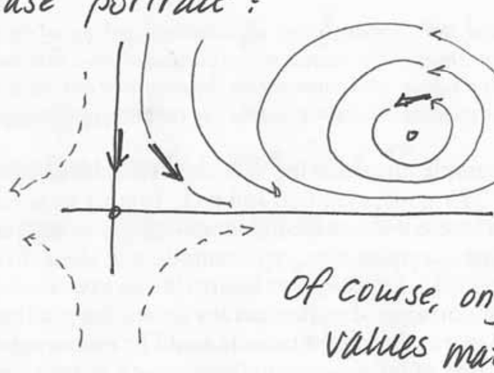
At $(80,40)$ $\quad \left.\begin{array}{l} \dot{D} = (30-40)(80) = -800 \\ \dot{w} = (.5(80)-40)40 = 0 \end{array}\right\}$ Vector points $\leftarrow$

At $(0,10)$ $\quad \left.\begin{array}{l} \dot{D} = (30-10)(0) = 0 \\ \dot{w} = (.5(0)-40)(10) = -400 \end{array}\right\}$ Vector points $\downarrow$

At $(10,10)$ $\quad \left.\begin{array}{l} \dot{D} = 200 \\ \dot{w} = -350 \end{array}\right)$ Vector points $\searrow$

HW # 3

The phase portrait:



Of course, only the positive values matter because we cannot have a negative population.