Final Exam Questions from 2006

1. (40 points) Consider a series RLC circuit being driven with an input voltage $v_i(t)$. The output voltage $v_o(t)$ is taken between the R and the C (together) .
   a) Give the expression for the (frequency space) voltage gain, $\mathbf{G}_v(\omega)$.
   b) Find $v_o(t)$ when $v_i(t) = 10 \sin(2t)$
   c) Find the value of ω_0 for which $v_i(t) = 10 \sin(\omega_0 t)$ is in phase with $v_o(t)$.
   d) If $v_i(t) = 10 \sin(3t) + 5 \cos(5t)$, are there any set of R, L, C values for which $v_o(t)$ can be in phase with $v_i(t)$? If not, why not?
   e) We use as input $v_i(t)$ a very sharp voltage spike of 1 V at $t = 0$, which we can approximate as a Dirac delta, $v_i(t) = \delta(t)$. Use Fourier concepts to find $v_o(t)$.
      - Hint: $\mathcal{F}[e^{-at} u(t)] = 1/(a+j\omega)$, where $u(t)$ is the step function (0 for all negative t and 1 for all positive t).
   f) Same as e) but with $v_i(t) = \delta(t) + \delta(t-10)$.

2. (30 points) Use Fourier concepts to explain why a 100 nm-wide speck of dust appears as a bright disk and a set of colorful concentric rings when observed with white light illumination and a microscope containing a cheap objective. How would you improve the objective and/or the lamp (if applicable) to try to image the speck more faithfully? (Note: your answer should address both size and color of the rings).

3. (30 points) You are shown for the first time a new kind of interferometer and you have to figure out how it works just by inspecting it and asking questions. First you see that there is an optical fiber connected to a light source and another optical fiber connected to a light detector (you trust the connectors work well, so that light is appropriately introduced into the fiber and the detector detects the light that exits the fiber). So far, so good. The two fibers seem to join into one a few centimeters down the fibers (it could be a meter, that distance is irrelevant). As a matter of fact, they are fused into one at that point – sort of like two roads (equally wide) merging into another road of the same width. Some distance down this junction, the fiber ends. We are told that this common end is polished so it forms a halfway mirror (50% of the light gets transmitted, 50% gets reflected back into the fiber). Indeed, when we shine the outcoming light into a different detector, the detector connected to the fiber optic reads about half the intensity of the detector outside of the fiber (the other half goes back into the light source). Thus, the optical fiber detector reads about one-fourth of the intensity injected into the fiber. So far, there is no interference. But when you place the end of the fiber really close (let's say a fraction of a millimeter) to a fully-reflective surface, you notice that sometimes the light read at the fiber optic detector decreases or increases, depending on the distance between the fiber end and the reflective surface. So, you (correctly) conclude that the waves reflected back at the half-mirror surface interfere with those that exited the fiber, got reflected at the fully-reflective surface and went back into the fiber. Very good! Not so fast …

   a) You are asked to provide the light source. Use Fourier concepts to compare the following light source(s) and the impact they have in the observed interference: 1) a white light bulb, 2) a light bulb covered with red paint (i.e. a red filter), 3) a light bulb covered with green paint (i.e. a green filter), 4) a green laser and 5) a red laser.

   b) For each of the above light sources, make an approximate graph (absolute scales irrelevant, but relative scale very relevant) of the light intensity at the detector as a function of distance between the fiber end and the reflective surface. In the same graph, plot what the curve would look like if you doubled the light intensity. <u>Note: if your graphs are messy, make sure it is clear which curve you consider final in your answer, or re-do the plot</u>.