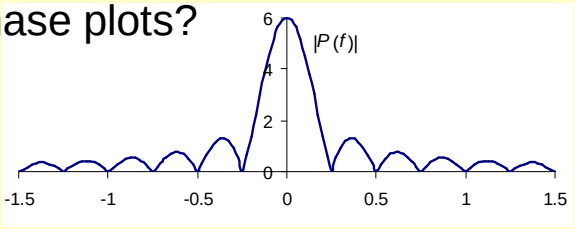
BIOEN 302

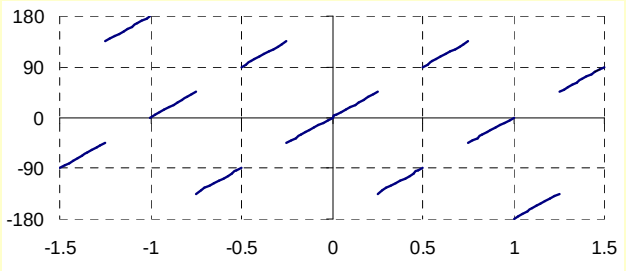
2007 Final Exam

Problem 3: Fourier Transforms

The question:

- What is the time-domain function that corresponds to the following magnitude and phase plots?

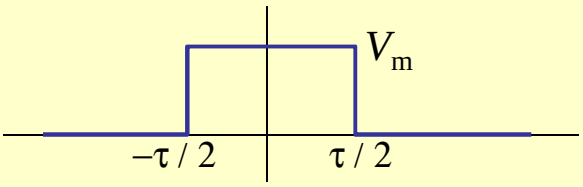


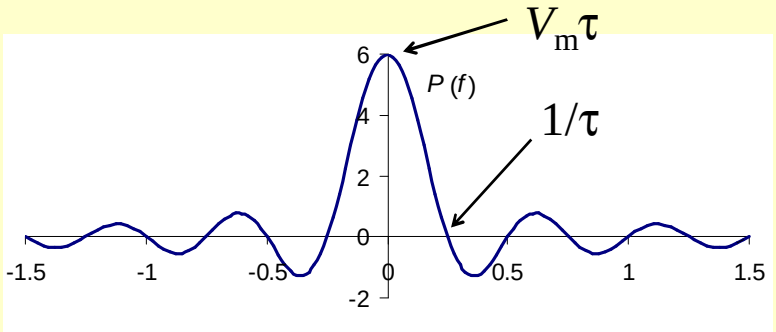


Thought process:

- The problem says that $|P(f)|$ is the absolute value of a sinc function
- We know that a sinc function is the Fourier transform (or inverse FT) of a square pulse

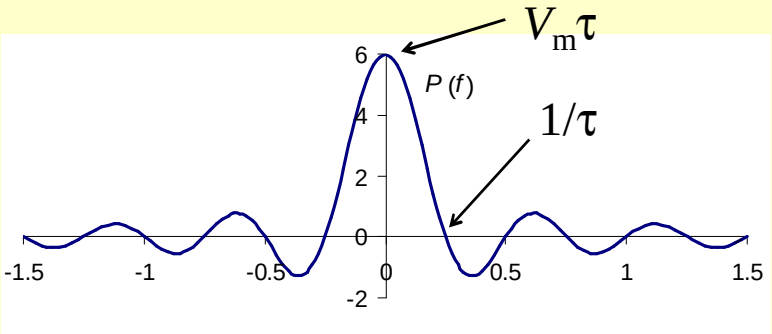
General form of pulse and sinc function





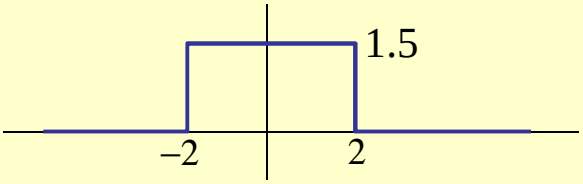
Quantitative solution

- Where do we get $1/\tau$ as the first minimum?
- $\sin(\pi f \tau)/(\pi f \tau) = 0$ when $\pi f \tau = \pi$



Finding Vm and τ

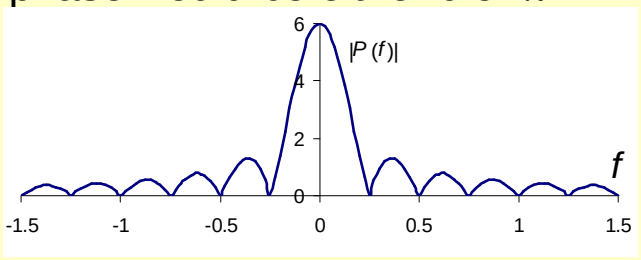
- $1/\tau = .25$ so $\tau = 4$
- $V_m\tau = 6$ so $V_m = 1.5$

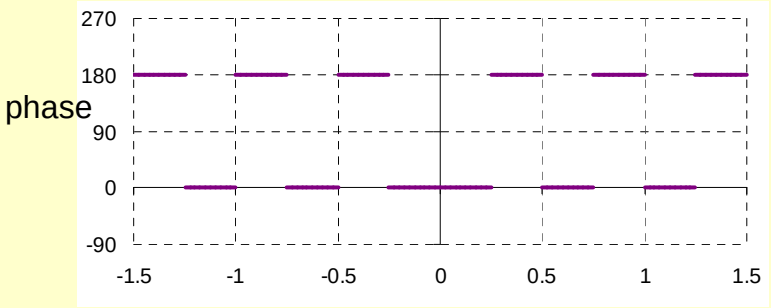


- We would be done here <u>if</u> the Fourier transform had been all real...
  but it is not.

Symmetric pulse -> real FT

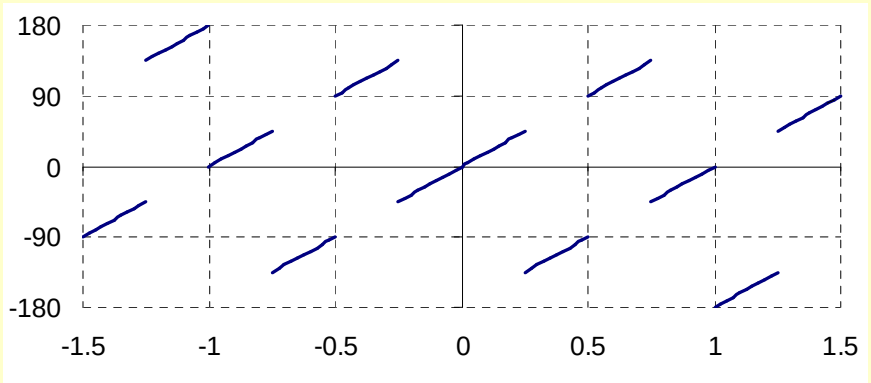
- The phase would be either 0 or π





What is the phase ϕ ?

- slope of ϕ = 45⁄.25 = π rad/Hz
- ϕ(*f*) = π*f* radians



What is the phase ϕ ?

- Let z be a complex number, z = a + jb
- z = Mcos(2pft – ϕ)
  where M = sqrt($a^2 + b^2$), ϕ = atan(b/a)
- z = M $e^{j\phi}$ (complex exponential form)
- Here, $P(f) = |P(f)| e^{j\phi} = |P(f)| e^{j\pi f}$
- But we know that
  $\mathcal{F}\{p(t-a)\} = \mathcal{F}\{p(t)\} e^{j2\pi fa}$
- Here, $\phi = \pi f = 2\pi fa$, so a = 1/2
- Therefore, the pulse is shifted right by 1/2

Final solution, part a

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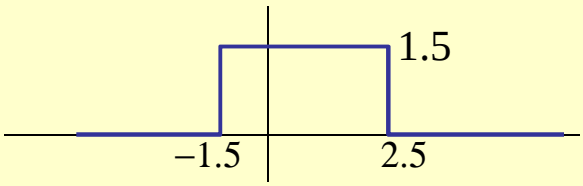
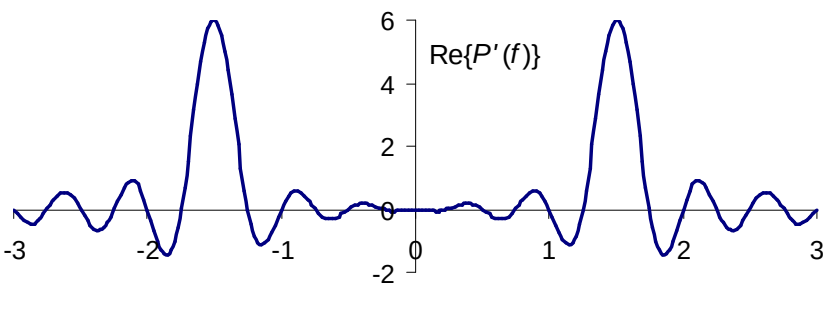


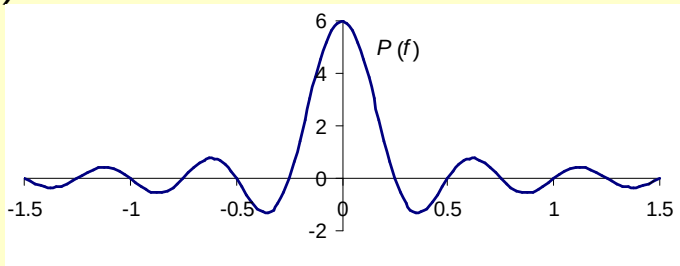
The next question:

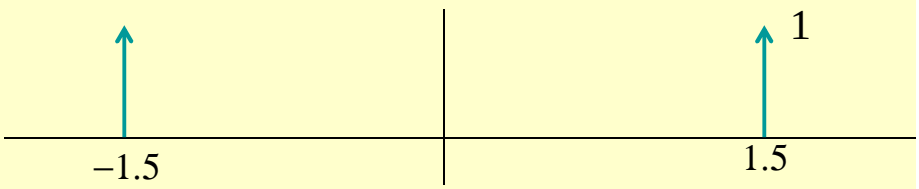
- What is the time-domain function with the FT that is all real, as shown here?



The next solution:

- $P(f)$ is the convolution of 2 functions





The next solution:

- We know the IFT of the first one already
- The second spectrum is the FT of $2\cos(2\pi f_0 t)$ where $f_0 = 1.5$
- Convolution in frequency means multiplication in time...
- So we get a *gated cosine*