

## Final Exam

December 8, 2008

**Permitted: Calculator, 2 bound books (including spiral notebooks – no loose papers)**  
**Not permitted: Electronic communication or data storage devices**

1. (24) When we discuss the eye, the term “spectrum” should be used carefully because it can mean a variety of things. More specifically, we can draw spectra with three fundamentally different units along the horizontal axis.

For each type of spectrum, provide the following:

- ☉ The units commonly used on the horizontal axis;
- ☉ The physical meaning of these units;
- ☉ Whether this spectrum is the result of a Fourier transform, and if so, whether it is one, two or three dimensional;
- ☉ Provide an example of information that might be displayed using each type of spectrum (pictures are welcome).

2. (25 points) In some ways, the retina in a mammalian eye is similar to the charge coupled device (CCD) in a digital camera: the rods and cones respond to the light that falls on them, creating an array of pixels that are interpreted by the visual cortex. However, the neural pathways that connect the rods and cones to the cortex perform key processing functions so the information that arrives at the cortex is not a simple pixel map. For example, studies have shown that the signal from an individual optoreceptor (i.e. rod or cone) interacts with signals from nearby receptors, including receptors that are farther away than just its nearest neighbors. Some of these nearby receptors have an inhibitory effect, while some have an additive effect.

a) Using concepts from Fourier optics, explain how this neural interaction could improve our visual acuity, compared to the case where the retina acts like a CCD chip.

b) Given the way that our eyes accommodate a wide range of light levels, what is a primary weakness of this processing system?

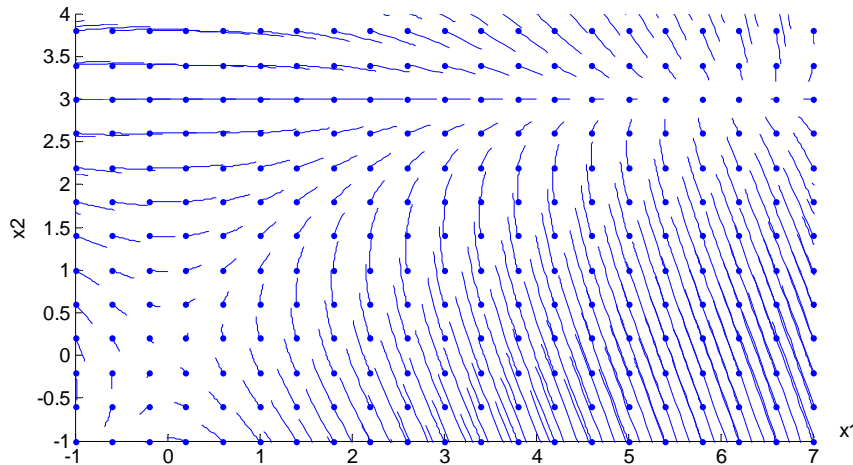
c) The principles that you have discussed above are also part of a process that allows a standard microscope to achieve performance similar to a confocal microscope. Briefly describe...

- ☉ This process
- ☉ The images that it creates
- ☉ How the system is fundamentally different from confocal microscopy
- ☉ How this process is similar to what occurs in the eye
- ☉ How this process is different from what occurs in the eye

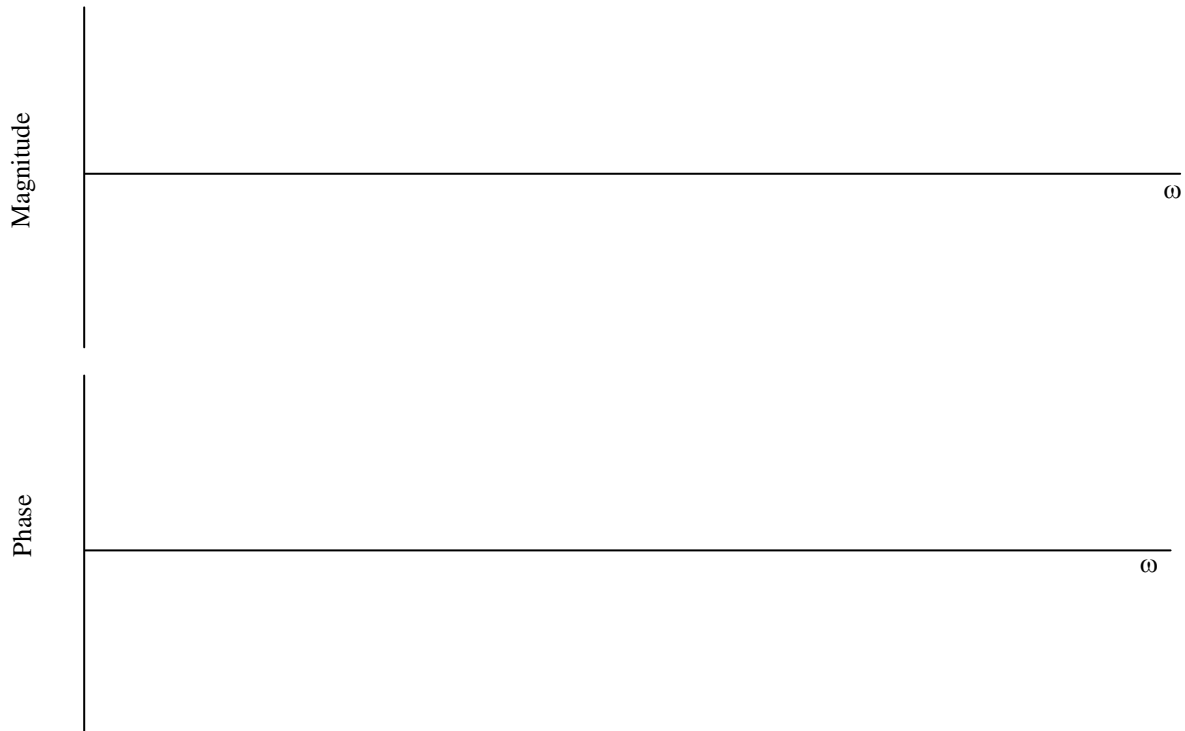
3. (25) Consider the following system of differential equations:

$$\begin{aligned}\dot{x}_1 &= 2x_2 - x_1 + 9u(t) \\ \dot{x}_2 &= x_1(3 - x_2)\end{aligned}$$

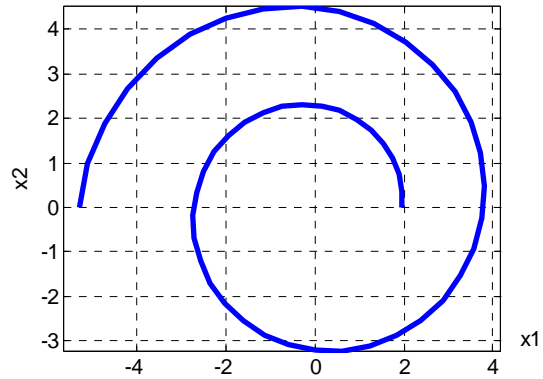
where  $u(t)$  is a system input, and the system output  $y$  is defined as  $10x_1$ . Its phase portrait shows two equilibrium points; one is a saddle and the other is a node.



- Why is it not possible to create a transfer function that represents the exact or complete behavior of this system?
- If we do create the one or more transfer functions based on this system, what should we assume about the values of the state variables  $x_1$  and  $x_2$ ?
- Choose one of the equilibrium points and determine the transfer function that relates the output  $y(t)$  to the input  $u(t)$ .
- Create and label approximate Bode magnitude and phase plots, indicating any break points and the slope of each asymptotic segment.



4. (26) A particular two state system has an equilibrium point at  $(x_1, x_2) = (0, 0)$ . The eigenvalues of the system are  $\lambda = 2 \pm 3j \text{ sec}^{-1}$ , such that its phase portrait includes a single spiral point.



a) Find the transfer function  $H(s)$  for this system, assuming that  $H(0) = 1$  and that the transfer function has no zeros.

b) Draw an approximate pole-zero plot for this transfer function.

c) Our next step will be to implement a controller. The resulting system should have poles that are not in the same location as the poles in  $H(s)$ . Why is this important?

d) Draw a feedback diagram that includes user input, system output, a controller with transfer function  $G(s)$ , and the system  $H(s)$ . You may put the controller in the feedback path or the forward path. Call the resulting input-output transfer function  $TF(s)$ .

e) Let  $G(s)$  be a PID controller, and select the PID gains such that the feedback system has three poles: two on the imaginary axis at  $\pm j5 \text{ sec}^{-1}$ , and one at  $-6 \text{ sec}^{-1}$ .

f) Draw a pole-zero plot for the transfer function of the feedback system.

*P.S. This isn't a very good controller, but it can be derived without a computer!*