 Name ___________________________

Final Exam

December 14, 2009

Closed book. Calculator and scratch paper are permitted.
Not permitted: Electronic communication or data storage devices

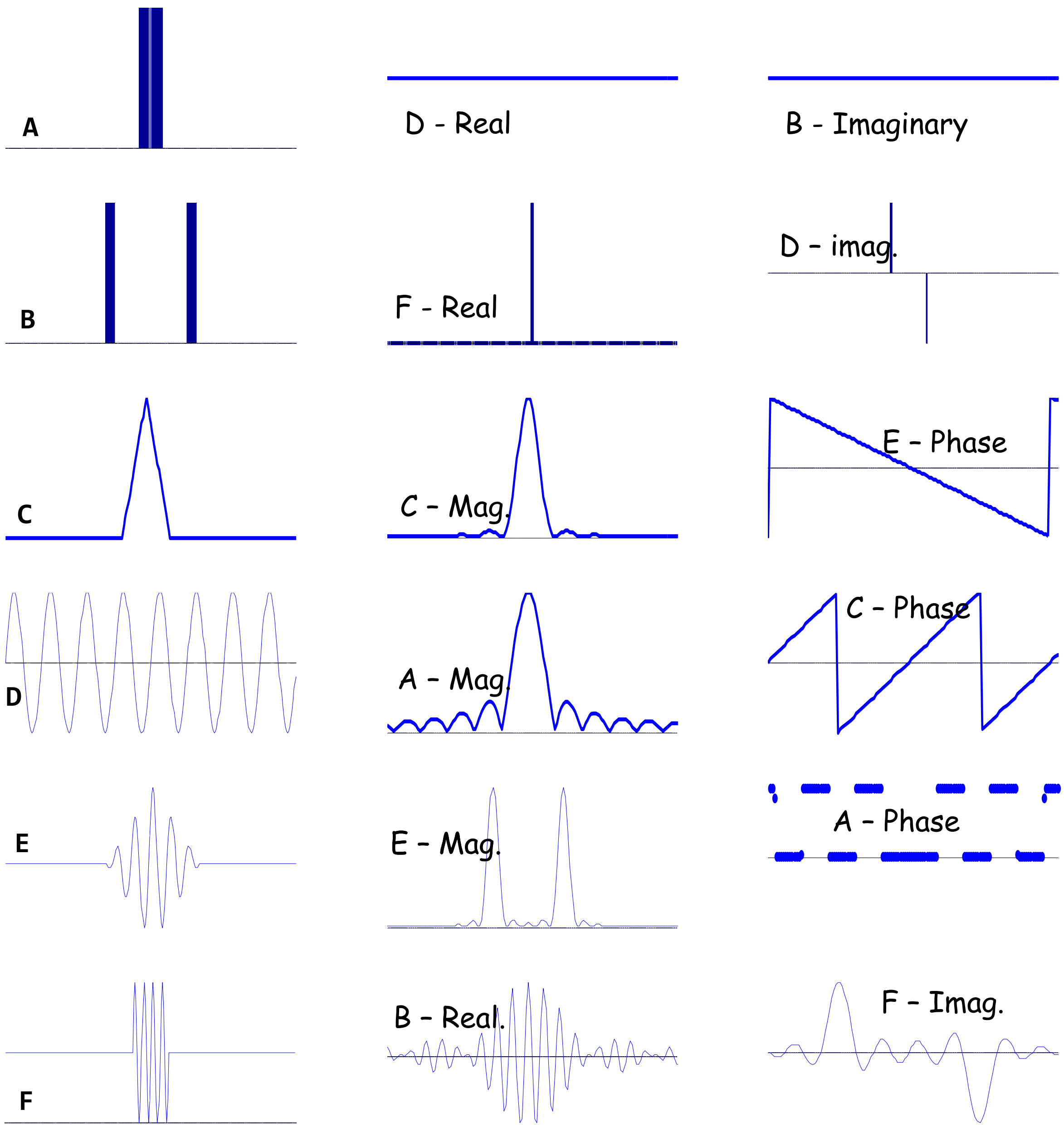
1. Discuss the following aspects of electrical isolation of hospital patients:
   The risks posed by unisolated equipment
   Safety features in the hospital electrical system
   Safety features in medical instruments

Please write in complete sentences and include labeled figures where appropriate.
(10 points for content, 10 points for clarity of explanation)

2. (20 points) In the following figure, the left column shows time domain plots, the middle column shows magnitude or real spectra, and the third column shows phase or imaginary spectra.

- The time domain axes all have the same time scale and are all centered on *t*=0.
- The frequency axes all have the same scale, and are centered on ω=0.
- The magnitude/phase/real/imaginary plots are randomly assorted so any one row probably does not contain the corresponding time and frequency plots.
- An arrow pointing horizontally indicates a time shift (some shifts are very small).

➢ For each frequency domain plot, write the letter of the corresponding time domain plot. Each letter should appear on one plot in the second column and one plot in the third column.
➢ For each frequency-domain plot, write Mag, Re, Im, or Ph, as appropriate.



3. (20 points) Suppose that you have collected step response data from a mechanical system, and used LabView to determine the open loop system transfer function, H(s). With options set to no zeros and two poles, the best fit comes out to be

$$H(s) = \frac{1}{s^2 + 6s - 27}$$

a) Comment on the system's stability. Illustrate your comments with one plot in the time domain and one plot in the Laplace (s) domain.

The system is unstable because there is one pole in the right half of the s-plane (at s = +3). The pole at s = −9 does not cause instability but it does not prevent it either. The Laplace-domain plot would show these two zeros. The time-domain plot would show an increasing exponential, or better yet the sum of an exp(+3t) term and an exp(−9t) term.

b) Implement a feedback control system to improve the system's stability.

Use only the proportional gain, k_p, such that G(s) = k_p. Put G(s) in the forward path.

Set the proportional gain to achieve critical damping of the controlled system.

Find TF(0).

Write the resulting transfer function, TF(s) = Y(s)/X(s).

Please see answers for parts (b) and (c) on the following pages.

c) If a constant forcing function is applied directly at the output (for example, ambient cooling of a heated object) then the proportional controller can exhibit a steady-state error.

Add a term to the controller gain G(s) that will eliminate the steady-state error.

Set the two gains to achieve critical damping of the controlled system.

Let TF(0) = 1.

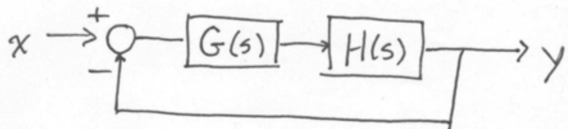
Write the resulting transfer function.

d) If G(s) from from part (c) is moved from the forward path to the feedback path, what is the new steady state gain, TF(0)?

e) Setting the gains in part (c) too high can cause overshoot in the system step response. What would you include in the G(s) to solve this problem, and basically how/why does it work?

Please see answer on following pages.

3 (b) If we put the controller $G(s)$ in the forward path:



Solve $Y(s) = GH \cdot (X - Y)$ to get $\frac{Y}{X} = \frac{G}{\frac{1}{H} + G}$

Let $G = k_p$

$$\frac{Y}{X} = \frac{k_p}{\frac{s^2+6s-27}{1} + k_p} = \frac{k_p}{s^2+6s+(k_p-27)}$$

To get critical damping we want repeated roots, which here means $\left(\frac{6}{2}\right)^2 = k_p - 27 \quad \therefore k_p = 36$

$$\frac{Y}{X} = \frac{36}{s^2+6s+9}$$

$$\frac{Y(0)}{X(0)} = 4$$

3(e) To eliminate overshoot we can add a derivative term. The faster the output is approaching the setpoint, the larger the derivative of the error will be. Including the derivative term effectively puts the brakes on the system as it approaches the setpoint, because the derivative is negative when the system is approaching the setpoint from below, ie the opposite sign compared to the proportional and integral terms.

3(c) To accommodate a steady-state influence we add an integral term.

Now $G(s) = k_p + \frac{1}{s}k_i = \frac{k_p s + k_i}{s}$

$$\frac{Y}{X} = \frac{G}{\frac{1}{H} + G} = \frac{1}{s}\,\frac{k_p s + k_i}{s^2 + 6s - 27 + \frac{k_p s + k_i}{s}}$$

$$= \frac{k_p s + k_i}{s^3 + 6s^2 - 27s + k_p s + k_i}$$

$$= \frac{k_p s + k_i}{s^3 + 6s^2 + (k_p - 27)s + k_i}$$

To get repeated roots for critical damping, we want the denominator in the form: $(s+a)^3$

i.e. $s^3 + 6s^2 + (k_p - 27)s + k_i = s^3 + 3s^2 a + 3sa^2 + a^3$

By inspection, $6s^2 = 3s^2 a \quad \therefore a = 2$

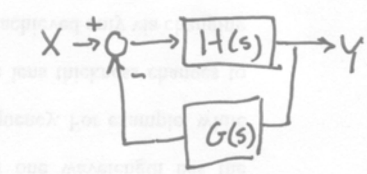
$k_i = a^3 \quad \therefore k_i = 8$

$k_p - 27 = 3a^2 \quad \therefore k_p = 3 \cdot 2^2 + 27 = 39$

so $\frac{Y(s)}{X(s)} = \frac{39s + 8}{s^3 + 6s^2 + 12s + 8}$

$$\left.\frac{Y}{X}\right|_{s=0} = 8/8 = 1.$$

3(d) Moving G to the feedback path...



means $\frac{Y}{X} = \frac{H}{1 + GH} = \frac{1}{\frac{1}{H} + G}$

Subsituting G and H gives

$$\frac{Y}{X} = \frac{1}{s^2 + 6s - 27 + \frac{k_p s + k_i}{s}} = \frac{s}{s^3 + 6s^2 + (k_p - 27)s + k_i}$$

$$\left.\frac{Y}{X}\right|_{s=0} = 0.$$

4. (20 points) Let us analyze the following network, in which X_0 and X_1 are fixed boundary species, k_n are the rate constants and S_n are the state variables.

X_0 -> S_1	k_1* X_0
S_1 -> S_2	k_2*S_1
S_2 -> X_1	k_3*S_2

a) Write the stoichiometry matrix, N, for this system.

b) Write out the differential equations for the two state variables.

c) Assume the following values for the constants:

$k_1 = 1$
$k_2 = 2$
$k_3 = 1$
$X_0 = 1$
$X_1 = 0$

From these values compute the steady state concentrations of S_1 and S_2.

d) Write out the equation for the complex gain (transfer function) for this system.

You might want to use the following formulas for matrix inversion:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{\begin{bmatrix} d & -c \\ -b & a \end{bmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}, \quad \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

state variable frequency response for this system.

e) Plot the magnitude and phase spectra for the system $[S_1\ S_2]^T$ with respect to a sinusoidal input, $X_0 = \cos(\omega t)$.

5. (20 pts)

a) Label each of the following operations as linear (L) or non-linear (N).

___N_Addition of a constant

___L_Multiplication by a constant

___L_Integration

___N_Exponentiation (square, cube, etc.)

___N_Phase angle calculation

_N/L_Addition of linear functions*

___L__Differentiation

___L__Fourier transformation

___N__Logarithmization

___N__Magnitude calculation

___L__Convolution with an impulse response

___L_Multiplication by an impulse response

___L__Multiplication by the Laplace transform of an impulse response

* *Depends on the function; question was not counted*

b) Think about the way that LTI systems modify a sinusoidal signal. Why does it make sense to describe the behavior of LTI systems using phasor notation?

Sinusoids are described by their frequency, amplitude, and phase (and sometimes by their mean value). If the input to an LTI system is a sinusoid, the output is also a sinusoid. The LTI system can modify the amplitude and phase, but not frequency. Phasor notation includes amplitude ratios and phase shifts explicitly, but the frequency is implicit, so it provides the information needed to describe the LTI system behavior.

c) What is the fundamental principle underlying Fourier series and transforms?

Any periodic signal can be expressed as an (infinite) sum of sinusoids, each of which has a frequency that is an integer multiple of some fundamental frequency. Any aperiodic signal can be represented as the integral of a frequency weighting function (implying that the fundamental frequency is zero).

d) Given a mass-spring-dashpot system, in which the spring and dashpot are in parallel, force applied to the mass is the input, and force along the dashpot is the output…

Write the transfer function in terms of m, k and b

Draw and label an electrical circuit that has the same transfer function.

What type of filter does this make?

$$TF(s) = \frac{\mathcal{L}\{F_{dashpot}\}}{\mathcal{L}\{F_{total}\}} = \frac{\mathcal{L}\{b\dot{x}\}}{\mathcal{L}\{m\ddot{x} + b\dot{x} + kx\}} = \frac{sbX}{s^2 mX + bsX + kX} = \frac{sb}{s^2 m + bs + k}$$

This is a band pass filter because TF $\rightarrow$ 0 as s $\rightarrow$ 0 and as s $\rightarrow \infty$.

The electrical equivalent is $TF = \dfrac{Rs}{s^2 L + Rs + 1/C}$ which can be generated by measuring voltage across a resistor in a series RLC circuit.