BIOEN 302
Autumn 2009

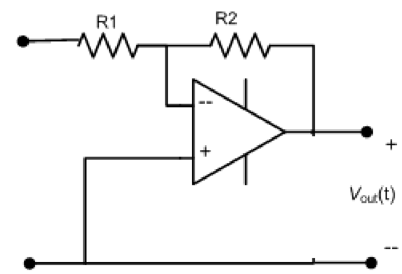
Name ____

Quiz 2 Practice

This was quiz 2 on October 21, 2009

Note: Throughout this quiz, you may leave your answers in terms of R_1, R_2, R_x and L, or you may use R_1 = 4 Ω, R_2 = 10 Ω, R_3 = 6 Ω, L = 2 H. [A table of Laplace transforms was provided on the original quiz but has been omitted from this practice sheet.]

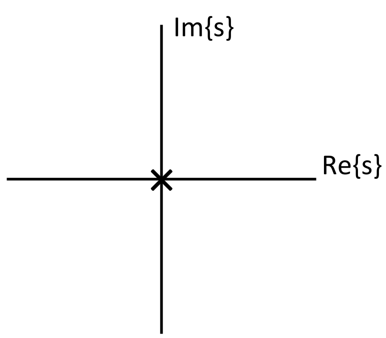
You know from EE 215 that, for an inverting op-amp circuit, $\frac{v_{out}}{v_{in}} = -\frac{R_2}{R_1}$.



a) Show that you can make an integrating circuit by replacing one of the resistors with an inductor. Steps to follow: State which resistor is replaced by the inductor, provide the transfer function for this circuit; write the Laplace transform pair that shows it is an integrator.

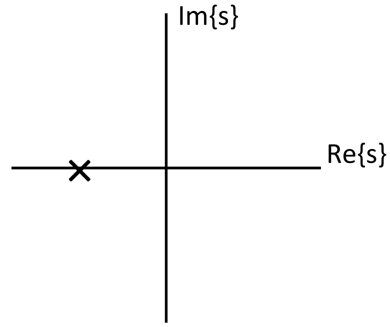
This is an inverting amplifier circuit, which as shown has a gain of −R2/R1. Replace R1 with an inductor, and work in impedances: Z_L = Ls, and Z_{R2} = R2. The transfer function Vout(s)/Vin(s) = −R2/Ls. The appropriate Laplace transform pair is $\int f(t)dt = \frac{F(s)}{s}$, where F(s) = −R2/L. Therefore it is an integrator. It is also a circuit with a unit impulse response that is a unit step; they are equivalent.

b) Mark any poles or zeros on the complex plane, and state whether the integrator is BIBO stable, unstable, or conditionally stable. Use the circuit's response to a unit impulse and its response to a unit step to support your choice.



There is one pole at 0,0 and no zeros, so the integrator is conditionally stable. When the input is a unit impulse, the output is a step with amplitude R2/L, which is finite. On the other hand, the response to a unit step is the function −(R2/L)t, which tends toward infinity as time increases. Therefore, whether the output is bounded depends on the input.

c) Show that you can insert a resistor R_x in the inductor-integrator circuit to make it more stable. It might not be as good at integrating, but that is OK. Steps to follow: draw the new circuit, provide its transfer function, mark a new pole-zero plot, and state whether it is now BIBO stable or conditionally stable.



To make the circuit stable we need to add a constant term in the denominator of the transfer function. That would make the denominator Ls+Rx. To get this sum we want the resistor in series with the inductor. The new transfer function is now

Vout/Vin = (-R2/L)/(s + Rx/L).

It has one pole at -Rx/L, and no zeros.

*Its impulse response is v(t) = (-R2/L)*exp(-t*Rx/L)*u(t), which integrates to -R2/Rx. If we integrate the absolute value we get +R2/Rx, which is bounded so it is bounded-input, bounded output (BIBO) stable.*

I have omitted the figure, but it has Rx and L in series where R1 was before, and R2 is unchanged.

$f(t)$	$F(s)$
$t^n u(t)$	$\frac{n!}{s^{n+1}}$
$\frac{d}{dt}f(t)$	$sF(s)$
$\int f(t)dt$	$\frac{F(s)}{s}$
$f(t-a)u(t-a)$	$e^{-as}F(s)$
$\int_{-\infty}^{+\infty} f(\lambda)g(t-\lambda)d\lambda$	$F(s)G(s)$