1 BIOEN 302: Review for Final Exam

1.1 Linear Time-invariant systems

- Linearity is indicated by

$$\begin{aligned} f(\alpha x) &= \alpha f(x) \text{ Scaling} \\ f(x_1 + x_2) &= f(x_1) + f(x_2) \text{ Superposition,} \end{aligned}$$

  where f is an arbitrary linear function and α is a constant.

- Time invariance means that if an input signal is shifted in time (delayed) but otherwise is unchanged, then the output will also be shifted by the same amount and will not be changed in any other way. Thus, the input and output can be functions of time but the transfer function cannot.

- The output from an LTI system can have a different amplitude and phase compared to the input, but if the input is a sinusoid then the output will also be a sinusoid of the same frequency.

- A differential equation of any order can be represented by a system of first-order differential equations.

- Systems of linear equations can be represented in state space form; systems of nonlinear equations cannot. However, nonlinear systems can be linearized around equilibrium points to show local behavior.

1.2 Frequency domain analysis

- Periodic functions can be represented by a (potentially infinite) sum of sinusoids and, therefore, by a Fourier series.

- Many aperiodic functions can be represented by a continuous distribution across a frequency spectrum, similar to a probability density function. This distribution can be found via the Fourier transform.

- Laplace transforms are better for handling transient signals than Fourier transforms are. Because the Laplace variable includes a real part, some functions that do not have a Fourier transform (due to lack of convergence) do have a Laplace transform.

Signals that are discrete in time are periodic in frequency.
Signals that are continuous in time are aperiodic in frequency.
Signals that are periodic in time are discrete in frequency.
Signals that are aperiodic in time are continuous in frequency.
Real signals have magnitude spectra that are even.
The real part of the complex spectrum is even and the imaginary part is odd.

- The broader a feature is in time or space, the narrower is its corresponding frequency spectrum.

- The ratio of complex voltage to complex current is the impedance. Impedance can be represented by a complex number or a phasor.

- Using impedances allows energy storage elements (L and C) to be treated as if they were all resistors, thus allowing RLC systems to be analyzed using algebra instead of differential equations.

- A transfer function is the ratio of output to input, both represented in the frequency domain (usually based on Laplace transforms). A transfer function is also the Fourier or Laplace transform of the system's unit impulse response.

- The magnitude or phase spectrum of a system can be found by setting $s = j\omega$ in its transfer function.

- Physical systems have features similar to RLC circuits, and hence they act as filters. Thus, "filter" and "system" can be used interchangeably. All we need to know is the transfer function.

- Shifting in time is equivalent to multiplying by a complex exponential in frequency. This should result in a phase plot that is a linear function of frequency. When a signal passes through a system that has a nonlinear phase plot, the output is a distorted version of the input.

- A transfer function can be obtained from a magnitude and phase spectrum and vice versa.

- Differential equations can be solved algebraically by working in the frequency domain:

  - Take the Fourier or Laplace transform of the input and system response
  - Multiply the input and transfer function
  - Use partial fractions to find the individual terms in the output
  - Use the inverse transform to find the time-domain output.

- A Bode plot is a way to estimate the frequency response of a system, given its transfer function. Each zero contributes an upward slope of 20 dB/decade; each pole contributes a downward slope of 20 dB per decade. A gain of 1 equals 0 dB. At the cutoff frequency (which is also the $\frac{1}{2}$ - power frequency), each pole contributes -3 dB.
- Convolution in the time (frequency) domain is equivalent to multiplication in the frequency (time) domain.

1.3 Fourier optics

- A far-field diffraction pattern is nearly equivalent to the 2-D Fourier transform of the aperture.
- A point spread function is the 2- or 3-D image created when the object is a single point.
- An image from an optical system (e.g. microscope) is the convolution of the ideal image with the point spread function of the optical system.

1.4 System controllers

- Open-loop control requires the user to supply the control signal for the system under control. Therefore the user must have accurate knowledge about the system under control.
- Closed-loop (feedback) control allows the input to be the desired output, allowing effective (although not optimal) control without accurate knowledge about the controlled system. This simplifies the machine-user interface.
- Poles are the locations in the complex plane where the denominator of a transfer function is equal to zero. When the real part of any of a system's poles is positive (the pole lies in the right half plane), that system is unstable.
- A feedback controller can make an unstable system stable, but can also introduce instability if applied incorrectly.
- A PID controller is put in the forward path of a feedback system. Purpose of PID control gains:
  - Proportional gain causes the output to track the input, but allows a steady-state error;

  - Integral gain eliminates SS error but increases overshoot or oscillations;
  - Derivative gain reduces overshoot.

1.5 Project management

- It takes longer to to do a project when you have to decide on and procure your own parts!
- LabView has a built-in function for just about every operation.
- You know more than you think you do.