BIOEN 302 Intro Biomed Instrumentation   Name: Answer Key

Quiz #4
November 5, 2007

1. (3 points) Convert the following differential equation into a system of first-order differential equations.

$$2y''' + 6y'' - 10y' + 14y = 8e^{-t}u(t)$$

$y = x_1$
$y' = x_2$
$y'' = x_3$

$\dot{x}_1 = x_2$
$\dot{x}_2 = x_3$
$\dot{x}_3 = -3x_3 + 5x_2 - 7x_1 + 4e^{-t}u(t)$

2. (3 points) Match each type of time-domain solution listed below to the corresponding type of critical point in a phase portrait.

__B__ Two decaying sinusoids
__G__ Two *non*-decaying sinusoids ($s = \pm j\omega$)
__F__ A critically damped system

Types of critical point:
A. Saddle point
B. Spiral point
C. Improper node
D. Proper node
E. Center
F. Focus
G. Sub-stable manifold

3. The number of people that get vaccinated against a certain disease depends both on the number of people that are not already vaccinated (they don't need it again), and the number of people that already have the disease (they create the fear that drives people to get vaccinated). Therefore, we might hypothesize the following system of equations:

$$\dot{V} = (D-2)(5-V)$$
$$\dot{D} = D(10-V)$$

where V is the number of people who have been vaccinated and D is the number of people who have the disease.

1. Find the equilibrium points for this system.
2. Linearize the system about an equilibrium point in which neither D nor V is zero.
3. Write the linearized system in state space form.
4. Find the eigenvalues for this system.

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You might wish to use the Jacobian matrix.

$$J = \begin{bmatrix} \frac{dy_1}{dx_1} & \frac{dy_1}{dx_2} \\ \frac{dy_2}{dx_1} & \frac{dy_2}{dx_2} \end{bmatrix}$$

(+3) 1) $0 = D(10-V) \Rightarrow D = 0$ or $V = 10 \Rightarrow \vec{x}_{eq1} = \begin{bmatrix} V \\ D \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$

$0 = (D-2)(5-V) \Rightarrow D = 2$ or $V = 5 \Rightarrow \vec{x}_{eq2} = \begin{bmatrix} V \\ D \end{bmatrix} = \begin{bmatrix} 10 \\ 2 \end{bmatrix}$

(+4) 2) $\vec{x}_{eq2}$ has non-zero V, D

$\dot{V} = 5D + 2V - 10 - DV$

$\dot{D} = 10D - DV$

using Jacobian matrix above $J = \begin{bmatrix} \frac{d\dot{V}}{dV} & \frac{d\dot{V}}{dD} \\ \frac{d\dot{D}}{dV} & \frac{d\dot{D}}{dD} \end{bmatrix}$

$J = \begin{bmatrix} 2-D & 5-V \\ -D & 10-V \end{bmatrix}$

using $\vec{x}_{eq2} = \begin{bmatrix} 10 \\ 2 \end{bmatrix}$

$A = \begin{bmatrix} 0 & -5 \\ -2 & 0 \end{bmatrix}$

(+4) 3) $\dot{\vec{x}} = \begin{bmatrix} \dot{V} \\ \dot{D} \end{bmatrix} = \begin{bmatrix} 0 & -5 \\ -2 & 0 \end{bmatrix} \vec{x}$ $\vec{x} = \begin{bmatrix} V \\ D \end{bmatrix}$

$\vec{y} = [1 \ \ 0] \vec{x}$

(+3) 4) $0 = |\lambda I - A|$

$0 = \begin{bmatrix} \lambda & 5 \\ 2 & \lambda \end{bmatrix}$

$\lambda^2 - 2(5) = 0$

$\lambda^2 = 10$

$\lambda = \pm\sqrt{10}$