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Final Exam

December 8, 2008

Permitted: Calculator, 2 bound books (including spiral notebooks – no loose papers)
Not permitted: Electronic communication or data storage devices

1. (24) When we discuss the eye, the term "spectrum" should be used carefully because it can mean a variety of things. More specifically, we can draw spectra with three fundamentally different units along the horizontal axis.

For each type of spectrum, provide the following:

- The units commonly used on the horizontal axis;
- The physical meaning of these units;
- Whether this spectrum is the result of a Fourier transform, and if so, whether it is one, two or three dimensional;
- Provide an example of information that might be displayed using each type of spectrum (pictures are welcome).

a) Electromagnetic spectrum

Units are in cm, nm, Å, etc.
Meaning is wavelength of light
Not the result of a FT
Absorption spectrum, emission spectrum

b) Temporal frequency spectrum

Units are typically 1/sec, radians/second, or Hertz
Meaning is the number of oscillations per unit time (if in Hertz) or the number of cycles times 2π (if radian frequency).
It is the result of a FT and will result in a Fourier series if the signal is periodic.
Examples: Magnitude and phase spectra in Bode plots, indicating the amplification and phase shift produced by a system (e.g. an RLC circuit) at any given frequency. Power spectrum in transmitted signal, e.g. from cell phone

c) Spatial frequency spectrum

Units are inverse length, e.g. 1/cm
Can be the result of Fourier transform in 1, 2 or 3 dimensions, although 2-D is most common. 1D FT's are actually degenerate 2D FT's where one dimension in space is so long that we can ignore its contribution in the frequency domain. 3D FTs can be used to describe point spread functions in microscope optics, but usually we just work with the PSF (in the space domain) because it is conceptually simpler.
Examples: Any image created by the FFT function in ImageJ; far-field diffraction patterns; frequency-domain representation of kernels used for image filtering (more on this in BIOEN 303).

2. (25 points) In some ways, the retina in a mammalian eye is similar to the charge coupled device (CCD) in a digital camera: the rods and cones respond to the light that falls on them, creating an array of pixels that are interpreted by the visual cortex. However, the neural pathways that connect the rods and cones to the cortex perform key processing functions so the information that arrives at the cortex is not a simple pixel map. For example, studies have shown that the signal from an individual optoreceptor (i.e. rod or cone) interacts with signals from nearby receptors, including receptors that are farther away than just its nearest neighbors. Some of these nearby receptors have an inhibitory effect, while some have an additive effect.

a) Using concepts from Fourier optics, explain how this neural interaction could improve our visual acuity, compared to the case where the retina acts like a CCD chip.

Relevant concepts: 1) Light passing through an aperture produces a far-field diffraction pattern that can be modeled as the 2-D Fourier transform of the aperture. The pattern we see is actually the square of the FT because we see the intensity of light rather than its amplitude. 2) Low spatial frequencies are represented by the pattern near the center of the diffraction. High spatial frequencies are represented by the outer part of a diffraction pattern. If the diffraction pattern extends to infinity then it contains enough information to reconstruct a perfect image of the object. 3) The propagation of light from a lens to an image is mathematically similar to an inverse Fourier transform. Therefore, the light from an object is effectively distributed in the frequency domain as it passes through a lens. 4) Blocking out part of the light before the lens focuses it as an image deletes some of the frequency content; blocking the outer parts deletes the high-frequency components. X) The pupil permits some of the light to pass through, effectively multiplying by a circle of value one within the pupil and by zero outside the pupil <u>in the frequency domain</u>. Y) multiplication in frequency is convolution in space. Z) the inverse FT of a circular aperture is a bessel function, which is a central maximum surrounded by rings. 5) Low spatial frequencies correspond to large areas in an image; high frequencies correspond to small features and edges. 6) Just as a lens can produce an image of an object that is some distance from the lens, a lens can produce a far-field diffraction pattern (of the lens aperture) relatively close to the lens – for example, at the back of the eye. 7) The image produced by an optical system is the convolution of the point spread function of the system and the image of the object.

The neural interaction could counteract the reduced visual acuity that occurs because of the point spread function within the eye, canceling out the rings that occur within any point. It is effectively deconvolving the Bessel function from the imperfect image.

b) Given the way that our eyes accommodate a wide range of light levels, what is a primary weakness of this processing system?

Our eyes accommodate changing light levels by contracting or dilating the pupil. The diameter of the Bessel function is inversely proportional to the width of the pupil, so the spacing of the inhibitory and additive neurons could be optimized for only one pupil diameter (or maybe some multiples of a fundamental diameter).

c) The principles that you have discussed above are also part of a process that allows a standard microscope to achieve performance similar to a confocal microscope. Briefly describe…

- This process
- The images that it creates
- How the system is fundamentally different from confocal microscopy
- How this process is similar to what occurs in the eye
- How this process is different from what occurs in the eye

The process is known as deconvolution microscopy. In it, a computer is used to reverse the convolution operation that occurs when light is focused through a lensing system of finite aperture (i.e. any system we could build).

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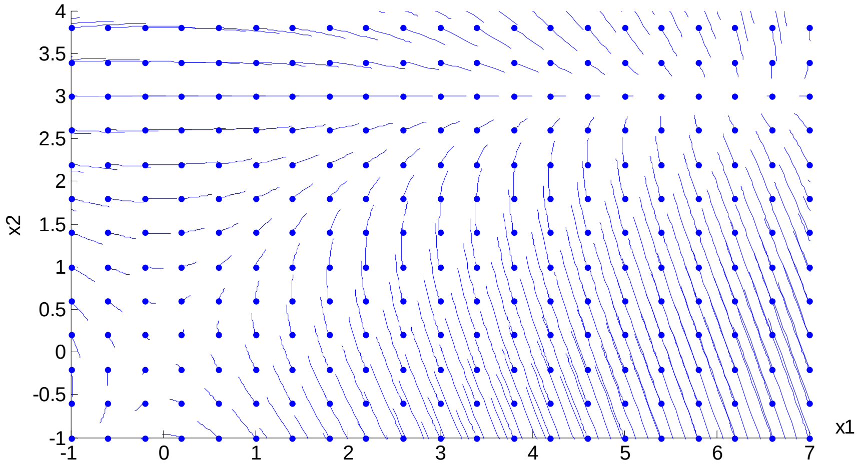
3. (25) Consider the following system of differential equations:

$$\dot{x}_1 = 2x_2 - x_1 + 9u(t)$$
$$\dot{x}_2 = x_1(3 - x_2)$$

where $u(t)$ is a system input, and the system output y is defined as $10x_1$. Its phase portrait shows two equilibrium points; one is a saddle and the other is a node.



Up to 3 points extra credit for phase portrait, not to exceed 25 points for the problem.

a) Why is it not possible to create a transfer function that represents the exact or complete behavior of this system?

4 points: the system is non-linear and we have defined transfer functions as LTI operations.

3 points: a transfer function can represent behavior around only 1 equilibrium point; this system has two. [A system with only one eq point can still be non-linear]

1 point: any true statement about systems of ODEs.

b) If we do create the one or more transfer functions based on this system, what should we assume about the values of the state variables x_1 and x_2?

3 points: Their values are close to the equilibrium point about which the system was linearized.

2 points: Their values do not change instantly (definition of state variable).

1 point: Any other true statement about state variables.

c) Choose one of the equilibrium points and determine the transfer function that relates the output $y(t)$ to the input $u(t)$.

d) Create and label approximate Bode magnitude and phase plots, indicating any break points and the slope of each asymptotic segment.

Solutions to parts c) and d) are provided in a separate file.

The solution to question 4 is in its own file, too.