BIOEN 302 : Possible solution to problem 3 from 2008 final exam.

Answers to (a) and (b) are shown on previously posted solution.

c) Equilibrium points occur where the user input is zero and the state derivatives are also zero. There are two pairs of states where this is true. You can find these by solving the equations; I did not pull them off of the phase portrait.

Case 1: x1 = 0, x2 = 0   Case 2: x1 = 6, x2 = 3

Transfer functions are linear and time invariant, so this non-linear system must be linearized around an equilibrium point before a transfer function can be generated. We use the Jacobian operator, which is a matrix of first derivatives that generate the first term in Taylor series. The derivatives are evaluated at each equilibrium point.
The Jacobian is defined as …

$J = \begin{bmatrix} \frac{d\dot{x}_1}{dx_1} & \frac{d\dot{x}_1}{dx_2} \\ \frac{d\dot{x}_2}{dx_1} & \frac{d\dot{x}_2}{dx_2} \end{bmatrix}$. Its output produces the state matrix A.

This state matrix is used in the following equation:

$$\dot{\bar{x}} = A\bar{x} + Bu(t)$$

$$y = C\bar{x}$$

In this system (the one in problem 3)

$$B = \begin{bmatrix} 9 \\ 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 10 & 0 \end{bmatrix}$$

The state matrix A depends on which equilibrium point you choose.

CASE 1 ⇒ $x_1 = 0,\ x_2 = 0$

$$A = \begin{bmatrix} -1 & 3 - x_2 \\ 2 & -x_1 \end{bmatrix}_{0,0} = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$$

A transfer function from a linear system is

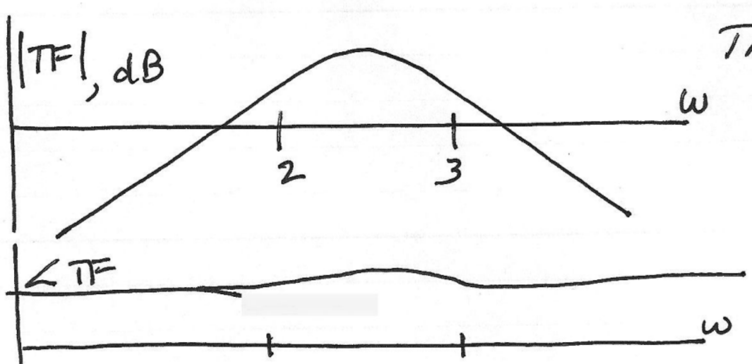
$$TF \equiv \frac{Y(j\omega)}{U(j\omega)} = C\left[j\omega I - A\right]^{-1} B$$

$$= \begin{bmatrix} 10 & 0 \end{bmatrix} \begin{bmatrix} j\omega+1 & 3 \\ 2 & j\omega \end{bmatrix}^{-1} \begin{bmatrix} 9 \\ 0 \end{bmatrix}$$

$$= \frac{\begin{bmatrix} 10 & 0 \end{bmatrix} \begin{bmatrix} j\omega & -2 \\ -3 & j\omega+1 \end{bmatrix} \begin{bmatrix} 9 \\ 0 \end{bmatrix}}{\begin{vmatrix} j\omega+1 & 3 \\ 2 & j\omega \end{vmatrix}}$$

$$= \frac{\begin{bmatrix} 10 & 0 \end{bmatrix} \begin{bmatrix} 9j\omega \\ -27 \end{bmatrix}}{j\omega(j\omega+1) - 6} = \frac{90\, j\omega}{(j\omega)^2 + j\omega - 6}$$

$$TF(j\omega) = \frac{90\, j\omega}{(j\omega+3)(j\omega-2)}$$



This is a band pass filter

The phase is nearly constant at +90°