BIOEN 302 Final Exam 2008 problem 4

Given:

Two-state system

Spiral point out of 0,0, with eigenvalues ʎ = 2±3j

Transfer function H(s) has no zeros

H(0) = 1.

Solution:

a) A two-state system has two poles, which are the eigenvalues and the roots of the characteristic equation. The general form of the transfer function is

$$H(s) = \frac{\beta}{(s-\lambda_1)(s-\lambda_1)}$$

$$H(s) = \frac{\beta}{(s-2-3j)(s-2+3j)} = \frac{\beta}{s^2-4s+13}$$

To make H(0)=1, β=13: $H(s) = \frac{13}{s^2-4s+13}$

b) The pole-zero plot shows the two eigenvalues on the complex plane… not difficult.

c) This system is unstable because its poles are in the right half plane, meaning that the real part of the roots to the characteristic equation are positive, so the impulse response will grow with time; we want it to decay. The next best thing is conditional stability, with the poles on the imaginary axis.

d) If we put a PID controller G(s) in the forward path, we get the overall transfer function…

$$TF(s) = \frac{GH}{1+GH} = \frac{1}{\frac{1}{H}+G}$$

e) Let $G(s) = \frac{k_i}{s} + k_p + k_d s = \frac{k_i + k_p s + k_d s^2}{s}$ and substitute H and G into TF(s). After algebra I get… $TF(s) = \frac{k_i + k_p s + k_d s^2}{s^3 + (13k_d - 4)s^2 + (13k_p + 13)s + 13k_i}$

The problem asks for three new poles, at ±5j and -6. This means that TF is

$$TF(s) = \frac{k_i + k_p s + k_d s^2}{s^3 + 6s^2 + 25s + 150}$$

Set the appropriate coefficients equal….

$(13k_d - 4) = 6$

$(13k_p + 13) = 25$ ...and solve for the three gains.

$13k_i = 150$

f) The new pole-zero plot has three poles which were given in part d, plus two zeros that are the roots of the numerator. I get z = -6±j*sqrt(114).