

Homework #3

Due in lecture, November 12

Predator Prey Model

Predator-prey models are commonly applied to ecological systems where there are two dominant species, one of which acts as herbivorous prey and the other as a carnivorous predator. In this case the system being modeled is the interaction of wolves and deer in an Alaskan wildlife preserve.

The model is as follows:

$$\dot{D} = (30 - W)D$$

$$\dot{W} = (.5D - 40)W - f$$

where D is the number of deer, and W is the number of wolves. Conservation ecologists may control the population of wolves in the preserve to ensure the health of the ecological system as a whole, which explains the input term, f .

Questions:

1. Find equilibrium points for this non-linear system;
2. Linearize the system about each equilibrium point to find the linear state matrices;
3. Rewrite the each linearized system in state space form;
4. Find the eigenvalues for each state matrix (*i.e.* the roots for each linearized system);
5. State whether the system will be stable, unstable, or conditionally stable near each equilibrium point;
6. Sketch the phase portrait of the nonlinear system.