A Review of Material You Should Have Learned Before Taking BIOEN 302

Mathematics

Logarithms

Let $\log(x) \equiv \log_{10}(x)$. Simplify the following expressions, if possible:

$\log(100x)$

$\log(x/\sqrt{10})$

$\log(100+x)$

$20\log(x^2)$

$20\log(x) - 10\log(1/x)$

Complex numbers

Label each of the following as real, imaginary, complex, or neither.

27	$\sqrt{-6}$
$4+5j$	$2e^{2j}$
$7e^{\pi j}$	e^{-j}
$2e^{-j\pi/2}$	$2e^{\pi/4}$
$e^{3j} + e^{-3j}$	$3e^{2j} + e^{2+5j}$

In the following questions let z* be the complex conjugate of the complex number z. Write an equivalent form for each of the following expressions:

$(3-2j)^*$

$(3-2j)^*(3-2j)$ i.e. the product of z and z*.

$(x+3-2j)^2$

Is $(x+3-2j)^*(x+3-2j)$ ….

a) Real

b) Imaginary

c) Complex

d) Depends on x

Add the following by converting to rectangular coordinates and back to polar.

$6e^{j\pi/6} + 2e^{-j\pi/6}$

Divide the following by converting to polar form and back.

$$\frac{2-3j}{7+4j} \qquad \frac{14+8j}{2} \qquad \frac{12j}{3-2j}$$

Partial derivatives

Obtain the following derivatives:

$$\frac{\partial}{\partial x}(x^2y - 4y^2) \qquad \frac{\partial}{\partial y}(x^2y - 4y^2)$$

Matrices

Let $A = \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 6 \end{bmatrix}$, $U = \begin{bmatrix} x \\ y \end{bmatrix}$

Find the following:

B·C

C·B

A·B

A·U+B

Write $\frac{d}{dx}\mathrm{U} = AU + B$ as two ordinary differential equations.

Conversely, you should know how to convert a system of ODEs into matrix form.

Differential Equations

First order ODEs

Find the general solutions for the following equations:

$$\dot{x}(t) = 2 - 3x$$

$$\dot{x}(t) + 2x = \sin(t)$$

Second-order ODEs

The following initial value problems are solved the same way, so finding solutions to all of them is rather repetitive. The point of including them here is to show how the class of step response (under, over or critically damped) can be modified by changing one term in the equation. Such modification is the basis of feedback control theory.

Examples to solve:

$$4x'' + 4x' + x = 12, \qquad x(0) = 0,\ x'(0) = 0$$

$$x'' + 7x' + 12x = 10, \qquad x(0) = 0,\ x'(0) = 0$$

$$x'' + 6x' + 5x = 10, \qquad x(0) = 0,\ x'(0) = 0$$

$$x'' + 6x' + 9x = 10, \qquad x(0) = 0,\ x'(0) = 0$$

$$x'' + 6x' + 13x = 10, \qquad x(0) = 0,\ x'(0) = 0$$

Note that the preceding IVPs have initial conditions equal to zero, because they are modeling a step input to a system at rest or equilibrium.

Laplace Transforms

Definition: $Y(s) = \mathcal{L}\{y(t)\} = \int_0^\infty e^{-st}\, y(t)dt$, where $s = \sigma + j\omega$.

You can think of s as the domain of all possible roots for a second-order ODE with constant coefficients.

A table of Laplace transforms may be found inside the front cover of the EE 215 book *Electric Circuits*, and in chapter 6 of the MATH 307 book (by Boyce and DiPrima).

One of the most important Laplace transform properties relates to differentiation:

$$\mathcal{L}\left\{\frac{d}{dt}y(t)\right\} = sY(s) - y(0)$$

$$\mathcal{L}\left\{\frac{d^2}{dt^2}y(t)\right\} = s^2Y(s) - sy(0) - y'(0)$$

Initial value problems are covered in section 6.2 of *Elementary Differential Equations*.

Suggested initial value problems for solution with Laplace transforms:

$$y'' + 6y' + 13y = 10, \qquad y(0) = 0,\ \ y'(0) = 0$$

$$y''(t) - y = 0,\ \ y(0) = 1,\ y'(0) = 2$$

$$\rho L\ddot{x} + \frac{8\mu L}{r^2}\dot{x} + 2\rho g x = e^{-t}\ \text{kPa}$$

$$\rho L = 1\ \text{kg/m}^2,\ \ \frac{8\mu L}{r^2} = 2\ \text{kg/m}^2\,\text{s},\ \ \rho g = 1\ \text{kg/m}^2\,\text{s}^2,$$

$$x(0) = 0,\quad \dot{x}(0) = 1\ \text{m/s}$$

Physics

Waves

Let x, y and z define the Cartesian coordinate system.

Traveling wave: $p = A\cos(kx - \omega t + \varphi)$, crests moving in the +$x$ direction

Stationary wave: $p = A\cos(kx + \varphi)$

Stationary oscillation: $p = A\cos(\omega t + \varphi)$

ω is in radians/second, $\omega = 2\pi f = 2\pi/T$, where f is frequency in Hz and T is period.

k is in radians/meter, $k = 2\pi/\lambda$, where λ is spatial period (wavelength).

Optics

Concepts: refraction, reflection, diffraction, concave and convex lenses, focal point, real and virtual images, index of refraction.

Formulas: Snell's law.

Engineering fundamentals

Heat transfer

Basic concepts: conduction, convection, radiation

Concepts in conductive heat transfer: thermal conductivity, diffusivity, and capacitance; heat capacity; lumped capacitance model.

Techniques: create COMSOL model of steady-state heat transfer; calculate heat flux in 1-dimensional thermal gradient, given ΔT and k.

Statics

Techniques: calculate deflection at the tip of a beam, assuming a load is concentrated at the tip of the beam.

Fluid mechanics

Concepts: fluidic resistance; identify factors that contribute to $\Delta P/Q$.

Techniques: Predict the flow rate as liquid is siphoned out of a container through a simple piece of tubing.

Electrical Engineering:

You should know how to do the following types of calculations:

Calculate the heat generated by a resistance. For example, determine how much heat would be produced in a wire, given its length, diameter, resistivity, and applied voltage.

Calculate the current through each resistor in a current divider, and the current through each resistor in a voltage divider.

Given a series or parallel RLC circuit and the internal voltages and currents at time t=0, derive the corresponding initial value problem.

Given a non-inverting op-amp circuit, calculate the circuit gain by starting with the ideal op-amp assumptions. See section 5.5 in *Electric Circuits* if you need a refresher.

Mechanical – electrical analogies

Match the following mechanical components or quantities with their electrical counterparts: dashpot, displacement, force, mass, spring, velocity.	
Capacitor	
Charge	
Current	
Inductor	
Resistor	
Voltage	

Given an electrical circuit, draw a mechanical schematic that would produce an equivalent differential equation.